Spring 2010 CSE310 Midterm Examination 02B (in class)

Instructions:

- There are five questions in this paper. Please use the space provided (below the questions) to write the answers.
- Budget your time to answer various questions and avoid spending too much time on a particular question.
- This is a **closed book** examination. You may not consult your books/notes.

NAME	
ASUID	

Problems	Score
P1	
P2	
P3	
P4	
P5	
Total	

P1. (20 points)

(10 pts) **This problem is related to order statistics.** What is the minimum number of element-wise comparisons needed to find the smallest element **and** the second smallest element in an un-ordered sequence of n elements? **Do not** use asymptotic notation in your answer.

Solution:

In order to find both the smallest and the second smallest elements in an array, $n + \lceil \log n \rceil - 2$ comparisons are needed.

First in $n/2$ comparisons, we have $n/2$ groups of 1^{st} and 2^0 candidate for 2^{nd} smallest in each group. In $n/4$ comparisons, we have $n/4$ groups of 1^{st} and 2^1 candidates for 2^{nd} smallest in each group. $\cdots$ In $n/2^k$ comparisons, we have $n/2^k$ groups of 1^{st} and 2^{k-1} candidates for 2^{nd} smallest in each group. In 1 comparison, we have 1 group of 1^{st} and $2^{\lceil \log n \rceil}$ candidates for 2^{nd} smallest in the group. We need $\lceil \log n \rceil - 1$ comparisons to find the 2^{nd} smallest element in the $2^{\lceil \log n \rceil}$ candidates. Thus, the total number of comparisons is $n + \lceil \log n \rceil - 2$.

Grading Keys:

5 pts for correct answer;
3 pts for the answer of the form $n + \log n + const$;
1 pts for the answer $2n - 3$.

Use the sequence

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8, 4, 7, 3, 6, 2, 5, 1
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to illustrate the process for finding the smallest element and the second smallest element. Show all the element-wise comparisons made for this example, in the correct order.

Solution:

The order of element-wise comparisons is as follows:
To find the smallest element: (8, 4), (7, 3), (6, 2), (5, 1), (4, 3), (2, 1), (3, 1)
To find the second smallest element: (3, 2), (2, 5)

Grading Keys:

+0.5 pts for each correct comparison.
0.5 bonus pts for all correct comparisons.

(10 pts) **This problem is related to the linear time selection algorithm.** Assume that we are using the linear time selection algorithm with group size 5. Suppose that we have 25 numbers and that they have been grouped into 5 groups of size 5 (separated by ;). $1, 2, 3, 4, 5$; $6, 7, 8, 9, 10$; $11, 12, 13, 14, 15$; $16, 17, 18, 19, 20$; $21, 22, 23, 24, 25$. What is the median of medians?

Solution:

The median of medians is 13.

Grading Keys:

5 pts for correct answer.

When we used groups of size 5, we have obtained the recurrence

$$T_5(n) \leq T_5(\lceil \frac{n}{5} \rceil) + T_5(\frac{7n}{10} + 6) + b \times n$$

for some constant b. What is the recurrence formula of the running time $T_7(n)$ for the corresponding select-k algorithm if we use groups of size 7.

Solutions: An important part of the derivation is to derive a lower bound of the *upper left portion* of the elements. There are at least $\lceil \frac{\lceil \frac{n}{7} \rceil}{2} \rceil - 2$ columns, each with at least 4 elements that are smaller than the median of medians. Therefore the *upper left portion* contains at least $\frac{4n}{14} - 8$ elements that are smaller than the median of medians. This leads to the recurrence

$$T_7(n) \leq T_7(\lceil \frac{n}{7} \rceil) + T_7(\frac{5}{7}n + 8) + bn.$$

Grading Keys:

5 pts for correct answer;

2 pts for including either the first or the second term of the recurrence;

1 pts will be cut off for incorrect const in the second term of the recurrence.