EENS 2110	Mineralogy
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External Symmetry of Crystals, 32 Crystal Classes

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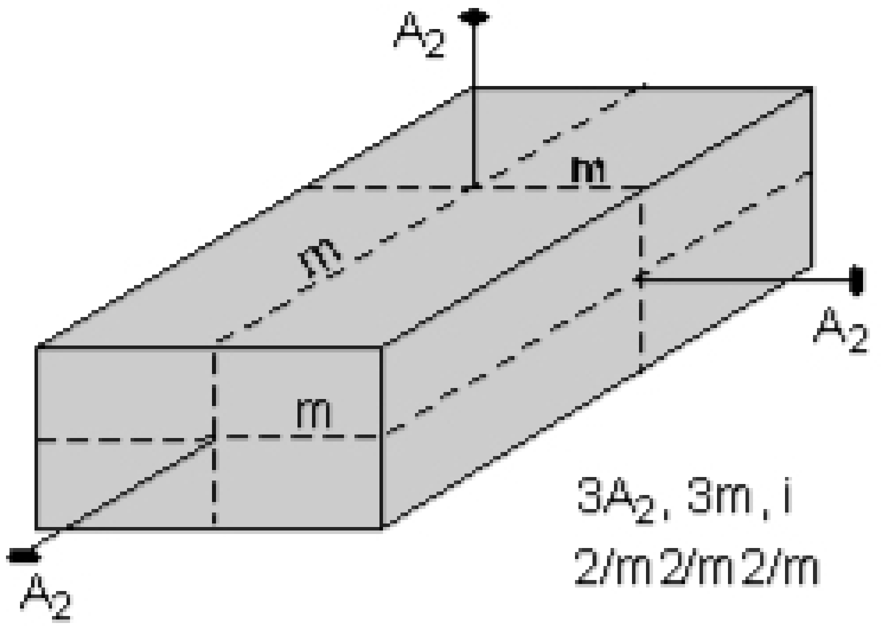
As stated in the last lecture, there are 32 possible combinations of symmetry operations that define the external symmetry of crystals. These 32 possible combinations result in the 32 crystal classes. These are often also referred to as the 32 point groups. We will go over some of these in detail in this lecture, but again I want to remind everyone that the best way to see this material is by looking at the crystal models in lab.

Hermann-Mauguin (International) Symbols

Before going into the 32 crystal classes, I first want to show you how to derive the Hermann-Mauguin symbols (also called the international symbols) used to describe the crystal classes from the symmetry content. We'll start with a simple crystal then look at some more complex examples.



The rectangular block shown here has 3 2-fold rotation axes (A_2), 3 mirror planes (m), and a center of symmetry (i). The rules for deriving the Hermann-Mauguin symbol are as follows:

1. Write a number representing each of the unique rotation axes present. A unique rotation axis is one that exists by itself and is not produced by another symmetry operation. In this case, all three 2-fold axes are unique, because each is perpendicular to a different shaped face, so we write a 2 (for 2-fold) for each axis

   2 2 2

2. Next we write an "m" for each unique mirror plane. Again, a unique mirror plane is one that is not produced by any other symmetry operation. In this example, we can tell that each mirror is unique because each one cuts a different looking face. So, we write:

   2 m 2 m 2 m

3. If any of the axes are perpendicular to a mirror plane we put a slash (/) between the symbol for the axis and the symbol for the mirror plane. In this case, each of the 2-fold axes are perpendicular to mirror planes, so our symbol becomes:

   2/m2/m2/m

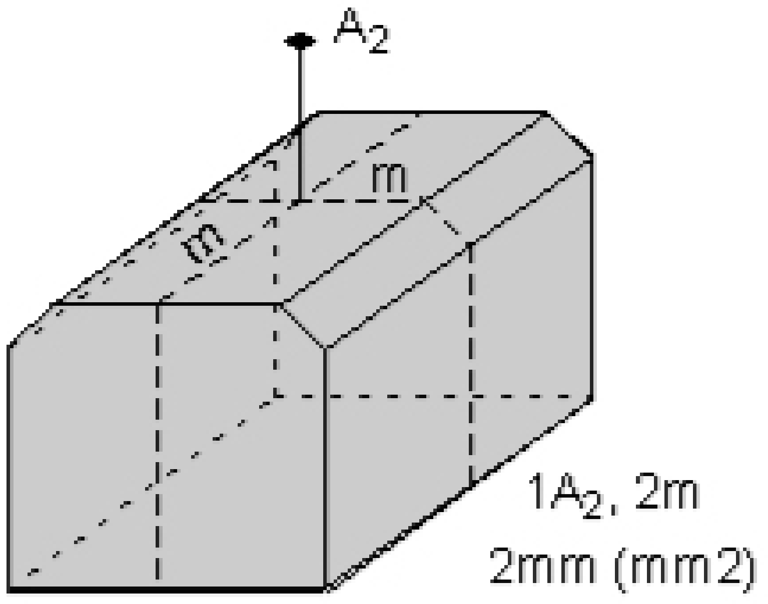
If you look in the table given in the lecture notes below, you will see that this crystal model belongs to the Rhombic-dipyramidal class.

Our second example involves the block shown here to the right. This model has one 2-fold axis and 2 mirror planes. For the 2-fold axis, we write:

2

Each of the mirror planes is unique. We can tell that because each one cuts a different looking face. So, we write 2 "m"s, one for each mirror plane:

2 m m



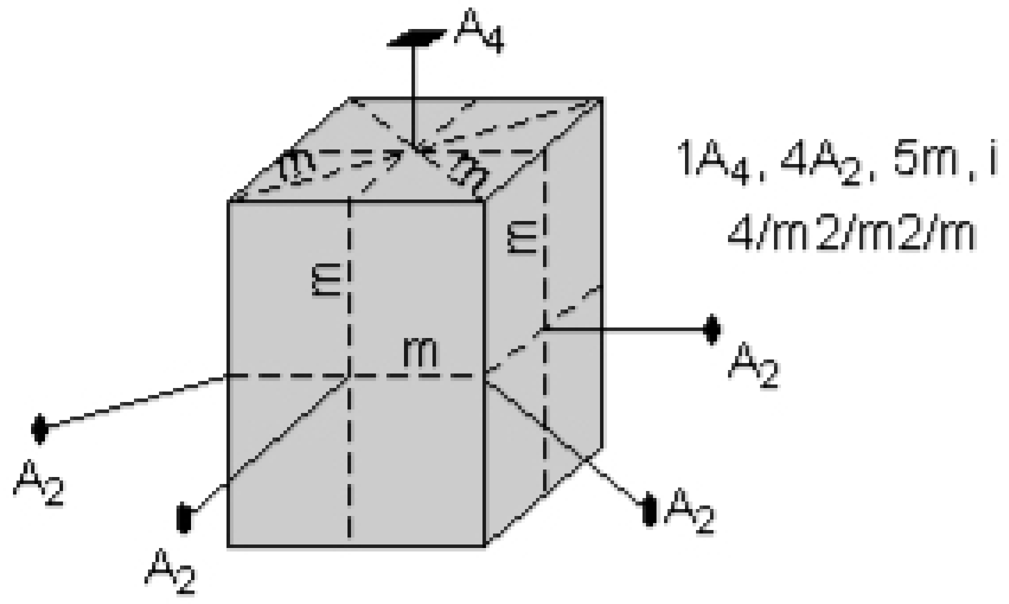
Note that the 2-fold axis is not perpendicular to a mirror plane, so we need no slashes. Our final symbol is then:

2mm

For this crystal class, the convention is to write mm2 rather than 2mm (I'm not sure why). If you consult the table below, you will see that this crystal model belongs to the Rhombic-pyramidal class.

The third example is shown here to the right. It contains 1 4-fold axis, 4 2-fold axes, 5 mirror planes, and a center of symmetry. Note that the 4-fold axis is unique. There are 2 2-fold axes that are perpendicular to identical faces, and 2 2-fold axes that run through the vertical edges of the crystal. Thus there are only 2 unique 2 fold axes, because the others are required by the 4-fold axis perpendicular to the top face. So, we write:

4 2 2

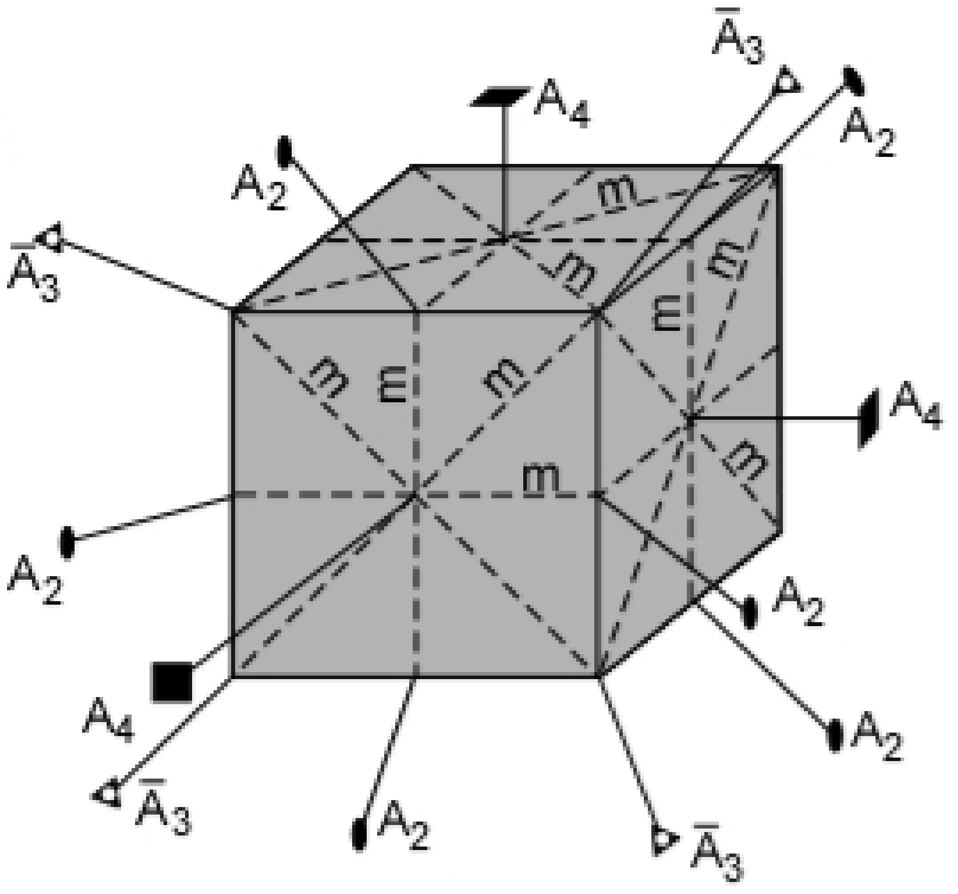


Although there are 5 mirror planes in the model, only 3 of them are unique. Two mirror planes cut the front and side faces of the crystal, and are perpendicular to the 2-fold axes that are perpendicular to these faces. Only one of these is unique, because the other is required by the 4-fold rotation axis. Another set of 2 mirror planes cuts diagonally across the top and down the edges of the model. Only one of these is unique, because the other is generated by the 4-fold rotation axis and the previously discussed mirror planes. The mirror plane that cuts horizontally through the crystal and is perpendicular to the 4-fold axis is unique. Since all mirror unique mirror planes are perpendicular to rotation axes, our final symbol becomes:

4/m2/m2/m

Looking in the table below, we see that this crystal belongs to the Ditetragonal-dipyramidal class.

Our last example is the most complex. Note that it has 3 4-fold rotation axes, each of which is perpendicular to a square shaped face, 4 3-fold rotoinversion axes (some of which are not shown in the diagram to reduce complexity), each sticking out of the corners of the cube, and 6 2-fold rotation axes (again, not all are shown), sticking out of the edges of the cube. In addition, the crystal has 9 mirror planes, and a center of symmetry.



$3A_4, 6A_2, 4\bar{A}_3$, 9m, i
$4/m\bar{3}2/m$

There is only 1 unique 4 fold axis, because each is perpendicular to a similar looking face (the faces of the cube). There is only one unique 3-fold rotoinversion axes, because all of them stick out of the corners of the cube, and all are related by the 4-fold symmetry. And, there is only 1 unique 2-fold axis, because all of the others stick out of the edges of the cube and are related by the mirror planes the other set of 2-fold axes. So, we write a 4, a $\bar{3}$, and a 2 for each of the unique rotation axes.

$$4\ \bar{3}\ 2$$

There are 3 mirror planes that are perpendicular to the 4 fold axes, and 6 mirror planes that are perpendicular to the 2-fold axes. No mirror planes are perpendicular to the 3-fold rotoinversion axes. So, our final symbol becomes:

$$4/m\bar{3}2/m$$

Consulting the table in the lecture notes below, reveals that this crystal belongs to the hexoctahedral crystal class.

The 32 Crystal Classes

The 32 crystal classes represent the 32 possible combinations of symmetry operations. Each crystal class will have crystal faces that uniquely define the symmetry of the class. These faces, or groups of faces are called crystal forms. Note that you are not expected to memorize the crystal classes, their names, or the symmetry associated with each class. You will, however, be expected to determine the symmetry content of crystal models, after which you can consult the tables in your textbook, lab handouts, or lecture notes. All testing on this material in the lab will be open book.

In this lecture we will go over some of the crystal classes and their symmetry. I will not be able to cover all of the 32 classes. You will, however, see many of the 32 classes during your lab work. Note that it is not easy to draw a crystal of some classes without adding more symmetry or that can be easily seen in a two dimensional drawing.

The table below shows the 32 crystal classes, their symmetry, Hermann-Mauguin symbol, and class name.