Pages 21-45 in Nicholson and Snyder

1. Mathematics for economics

   1.1. Much of the math used in microeconomics is focused on finding the optimal value of some variable

      1.1.1. "Optimal" being the value that maximizes (or minimizes) some objective

      1.1.2. Maximization/minimization of functions

   1.2. The portions of this chapter that we skip involve

      1.2.1. Identification of certain properties of functions that are useful in building economic models

      1.2.2. Techniques to maximize a stream of objectives over time

      1.2.3. Mathematical statistics, which is used when an economic model includes uncertainty regarding one or more variables

   1.3. This isn't a math class, but, since our economic models are built using mathematics, it is necessary to understand it (the same way that knowledge of the English language is necessary for understanding of the models I describe)

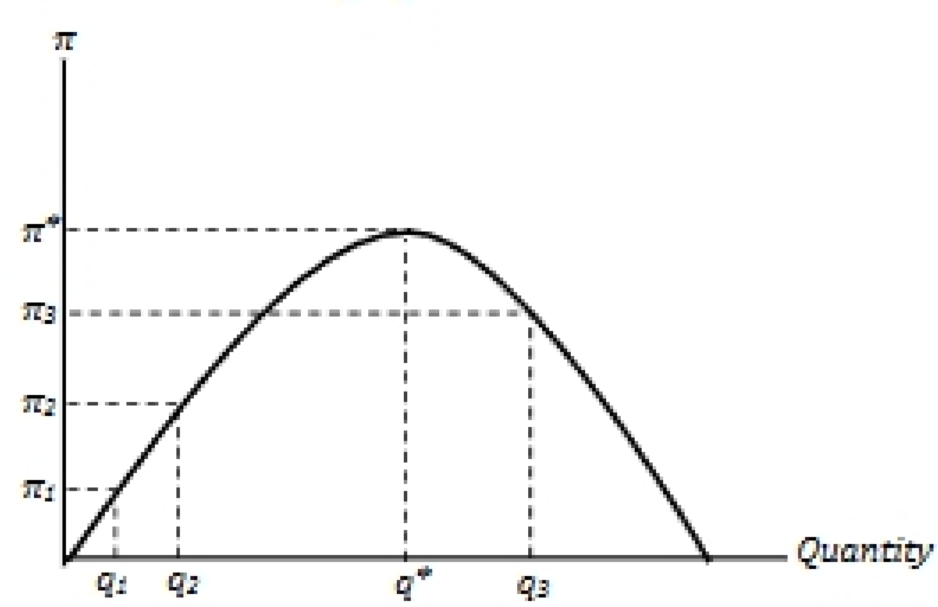
2. Maximization of a function of a single variable

   2.1. Example: suppose that a manager wants to maximize the profits received from selling a particular good

      2.1.1. Profits received (π) depend on the quantity of the good sold (q)

$$\pi = f(q) \tag{1}$$

      2.1.2. If $f(q)$ is known, we could graph it and find its maximum and optimal q^*



2.1.3. If $f(q)$ isn't known, the manager could try changing the value of q to see what happens to profit

2.1.3.1. Say the manager starts with $q = q_1$ and $\pi = \pi_1$; next the manager tries increasing output to q_2, and observes that profit increases to π_2

2.1.3.2. Profits have increased in response to an increase in q, or

$$\frac{\pi_2 - \pi_1}{q_2 - q_1} > 0, \text{ or } \frac{\Delta\pi}{\Delta q} > 0 \tag{2}$$

2.1.3.3. As long as ${}^{\Delta\pi}/_{\Delta q} > 0$, profits increase as the manager increases output

2.1.3.4. This occurs until $q > q^*$, in which case increasing output decreases profit, or ${}^{\Delta\pi}/_{\Delta q} < 0$

2.2. The limit of ${}^{\Delta\pi}/_{\Delta q}$ as Δq goes to zero is called the *derivative* of the function $\pi = f(q)$, and is denoted $d\pi/dq$, df/dq or $f'(q)$

2.2.1. The formal definition of the derivative of f at point q_1 is

$$\frac{d\pi}{dq} = \frac{df}{dq} = \lim_{h\to 0}\frac{f(q_1 + h) - f(q_1)}{h} \tag{3}$$

2.2.2. We can denote the value of a derivative at a specific point like this:

$$\frac{d\pi}{dq}\Big|_{q=q_1} \tag{4}$$

2.2.2.1. So, for the example above, we could say that $\frac{d\pi}{dq}\big|_{q=q_1} > 0$ and that $\frac{d\pi}{dq}\big|_{q=q_2} < 0$

2.2.2.2. What is the value of $\frac{d\pi}{dq}\big|_{q=q^*}$?

2.3. First-order condition for a maximum: for a function of 1 variable to be maximized at some point, the value of the derivative of that function at that point must be zero

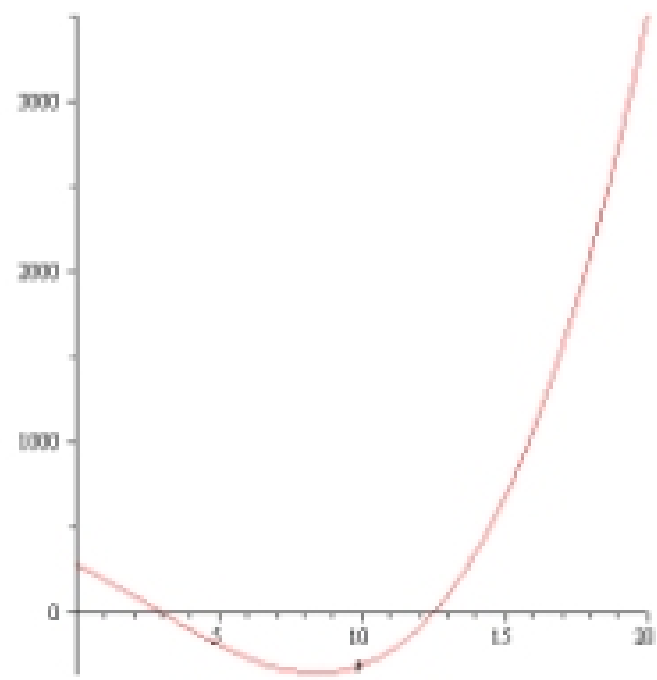
$$\frac{d\pi}{dq}\Big|_{q=q^*} = 0 \tag{5}$$

2.3.1. Why is this true? What would it mean if ${d\pi}/{dq} > 0$? ${d\pi}/{dq} < 0$?

2.4. Second-order conditions

2.4.1. It's possible for the derivative of a function to be zero at a certain point, but not be maximized at that point

2.4.1.1. What is the value of the derivative at the minimum of a function?



2.4.1.2. What is the value of the derivative at $x = 14$ below?