ROTATIONAL MOTION

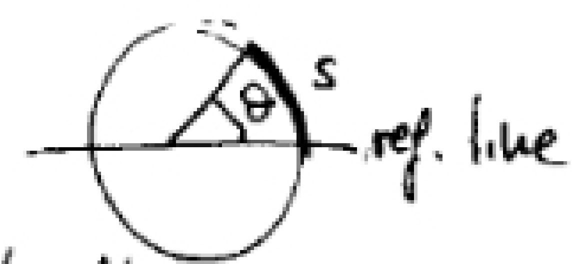
this chapter deals with rotational motion and all definitions and applications are equivalent to what we have seen in chapter 2 (1-D kinematics) that we should now call: "translational kinematics" to differenciate from "rotational kinematics".

As we will see, the equations of rotational motion have the same form than the equations of translational motion, by just changing "x" by "θ", "v" by "ω" and "a" by "α", where:

ANGULAR POSITION, VELOCITY AND ACCELERATION

1) θ is the angle measured from a reference line

$[\theta] = \text{rad}$



$\boxed{\begin{array}{c}\theta > 0 \\ \text{counterclockwise}\end{array}}$

and "s" is the arc length

Note that $s = r\theta$

* so 1 radian is the angle for which the arc length "s" on a circle of radius "r" is equal to the radius.
* One revolution, $1\,\text{rev} = 2\pi\,\text{rad}$.
* [rad] is dimensionless.

We define angular displacement as: $\Delta\theta = \theta_f - \theta_i$

2) ω is the angular velocity, and is defined as:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad [\omega_{av}] = s^{-1} \equiv \frac{rad}{s}$$

in the same way we did with linear velocity "v" we can also define instantaneous angular velocity "ω" as:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$\omega > 0$ counterclockwise rotation

time to complete a revolution (Period)

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} \longrightarrow T = \frac{2\pi}{\omega} \qquad [T] = s$$

3) α is the angular acceleration.

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$ and instantaneous angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad [\alpha] = \frac{rad}{s^2} = s^{-2}$$

$$\alpha = \frac{d\omega}{dt}$$

EQUATIONS OF ROTATIONAL MOTION

Once we have found the equivalence between positions, velocities and accelerations, we can obtain the equations of motion for rotation by direct translation of the equations of translational motion seen in chapter 2.

As a reminder (for constant a, α)

$$\left.\begin{aligned} x &= x_0 + v_0 t + \tfrac{1}{2} a t^2 \\ v &= v_0 + a t \end{aligned}\right\} \text{Translational motion}$$

$$\left.\begin{aligned} \theta &= \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2 \\ \omega &= \omega_0 + \alpha t \end{aligned}\right\} \text{Rotational motion}$$

ANGULAR AND TRANSLATIONAL QUANTITIES

<u>Tangential velocity</u> $\underline{v_t}$:

$$v_t = \frac{\Delta x}{\Delta t} = \frac{\text{perimeter}}{1 \text{ period}} = \frac{2\pi r}{T} = \frac{\cancel{2\pi} r}{\cancel{2\pi}/\omega} = r\omega$$

so $\boxed{v_t = r\omega = r\,\frac{2\pi}{T}}$