Calc III

Note-taker needed: http://get.studysoup.com/bardo-academic-access/

Preview cheese → cocheese

$f: \mathbb{R} \to \mathbb{R}$

$x \mapsto f(x)$

1) Derivatives and linear approximations

$f(a+h) = f(a) + f'(a)h + E(a,h)$

↑ Error

where $\frac{|E(a,h)|}{|h|} \to 0$ as $h \to 0$

Ex) $f(x) = x^3$

$f(5+h) = f(5) + f'(5)h + E(5,h)$

$(5+h)^3 = 5^3 + 3 \cdot 5^2 h + 3 \cdot 5h^2 + h^3$

$E(5,h) = 3 \cdot 5h^2 + h^3 \Rightarrow \frac{|E(5,h)|}{|h|} = \frac{|3 \cdot 5h^2 + h^3|}{|h|}$

$= |3 \cdot 5h + h^2| \to 0$ as $|h| \to 0$

2) Chain Rule

$(f \circ g)'(x) = f'(g(x))\, g'(x)$ or $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$

3) Integrals (Riemann Sums)

4) Antiderivatives

$f(x) = x^3$ $\int f(x)\,dx = \frac{1}{4}x^4 + C$

5) Fundamental Theorem of Calculus

$\int_a^b f(x)\,dx = f(b) - f(a)$

Multivariable Calculus

$f: \mathbb{R}^n \to \mathbb{R}^m$ domain $\mathbb{R}^n$ codomain $\mathbb{R}^m$

EX1) $f: \mathbb{R} \to \mathbb{R}^3$

$t \mapsto \langle t^2+4, \sin(t), e^{t^2} \rangle$ (Chapter 13)

$f(0) = \langle 4, 0, 1 \rangle$

EX2) $f: \mathbb{R}^3 \to \mathbb{R}$

$(x,y,z) \mapsto (x^2y - 3(y+2))z^3$ (Chapter 14)

$f(3,1,2) = f(\langle 3,1,2 \rangle)$

$= (9 \cdot 1 - 3(1+2))8$

$= \boxed{-63}$

EX3) $f: \mathbb{R}^3 \to \mathbb{R}^3$

$\langle x,y,z \rangle \mapsto \langle xy, 2yz, 4xyz \rangle$ (Chapter 16)

$f(2,3,5) = \langle 6, 30, 120 \rangle$

Chapter 13 8/26/14

Section 1: Vector valued functions

$r: \mathbb{R} \to \mathbb{R}^3$

$t \mapsto \langle x, y, z \rangle$ $\qquad x\hat{i} + y\hat{j} + z\hat{k}$

only when doing cross products.

EX) $r: \mathbb{R} \to \mathbb{R}^3$

$t \mapsto \langle t^3, t^2+4, 3t^2-1 \rangle$

$x(t) = t^3$

$y(t) = t^2+4$

$z(t) = 3t^2-1$

$\frac{dr}{dt}$ or $r'(t) = \langle \frac{d}{dt}(t^3), \frac{d}{dt}(t^2+4), \frac{d}{dt}(3t^2-1) \rangle$

$r'(t) = \langle 3t^2, 2t, 6t \rangle$

* $r(t+h) = r(t) + r'(t)h + E(t,h)$ $\quad$ where $\frac{|E(t,h)|}{|h|} \to 0$ as $|h| \to 0$

$r(t+h) = \langle (t+h)^3, (t+h)^2+4, 3(t+h)^2-1 \rangle$

$= \langle t^3 + 3t^2h + 3th^2 + h^3, t^2 + 2th + h^2 + 4, 3(t^2+2th+h^2)-1 \rangle$

$3t^2 + 6th + 3h^2 - 1$

$E(t,h) = r(t+h) - r(t) - r'(t)h$

$= \langle \cancel{t^3} + \cancel{3t^2h} + 3th^2 + h^3 - \cancel{t^3} - \cancel{3t^2h},$

$\cancel{t^2} + \cancel{2th} + h^2 + \cancel{4} - \cancel{(t^2+4)} - \cancel{2th},$

$\cancel{3t^2} + \cancel{6th} + 3h^2 \cancel{-1} - \cancel{(3t^2-1)} - \cancel{6th} \rangle$

$= \langle 3th^2 + h^3, h^2, 3h^2 \rangle$ $\quad$ (error term: $E(t,h)$)

$= h^2 \langle 3t+h, 1, 3 \rangle$

* $|v| = \sqrt{x^2+y^2}$ $\qquad |\alpha v| = |\alpha|\,|v|$

scalar vector abs value length

$|E(t,h)| = h^2\sqrt{(3t+h)^2 + (1)^2 + (3)^2}$

$\frac{|E(t,h)|}{|h|} = |h|\sqrt{(3t+h)^2 + (1)^2 + (3)^2} = |h|\sqrt{(3t+h)^2+10} \to 0$

as $|h| \to 0$