

Today's topics

- Relations
 - Kinds of relations
 - n-ary relations
 - Representations of relations
- Reading: Sections 7.1-7.3
- Upcoming
 - Minesweeper

Binary Relations

- Let A, B be any sets. A *binary relation* R from A to B , written (with signature) $R:A \times B$, or $R:A, B$, is (can be identified with) a subset of the set $A \times B$.
 - E.g., let $< : \mathbf{N} \leftrightarrow \mathbf{N} \equiv \{(n, m) \mid n < m\}$
- The notation $a R b$ or aRb means that $(a, b) \in R$.
 - E.g., $a < b$ means $(a, b) \in <$
- If aRb we may say “ a is related to b (by relation R)”,
 - or just “ a relates to b (under relation R)”.
- A binary relation R corresponds to a predicate function $P_R:A \times B \rightarrow \{\mathbf{T}, \mathbf{F}\}$ defined over the 2 sets A, B ;
 - e.g., predicate “eats” $\equiv \{(a, b) \mid \text{organism } a \text{ eats food } b\}$

Complementary Relations

- Let $R:A,B$ be any binary relation.
- Then, $\overline{R}:A\times B$, the *complement* of R , is the binary relation defined by
$$\overline{R} := \{(a,b) \mid (a,b) \notin R\} = (A\times B) - R$$
- Note this is just \overline{R} if the universe of discourse is $U = A\times B$; thus the name *complement*.
- Note the complement of \overline{R} is R .

Example: $\overline{\lt} = \{(a,b) \mid (a,b) \notin \lt\} = \{(a,b) \mid \neg a < b\} = \geq$