

Lecture 10: Fixed Points ad Infinitum

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Goal: Understand the Least Fixed Point Theorem

If D is a *pointed complete* partial order, then a *continuous function* $f: D \rightarrow D$ has a least fixed point ($\mathbf{fix}_D f$) defined by

$$\bigsqcup_D \{ (f^n \perp_D) \mid n \geq 0 \}$$

Last Time

- A *domain* is a structured set of values
- A *function domain* is constructed from two primitive domains, $D_1 \rightarrow D_2$ by associating an element of D_2 with each element of D_1 .
- A *fixed point* of a function $f: D_1 \rightarrow D_2$ is an element $d \in D$ such that $f d = d$.