Lecture 10:
Fixed Points ad Infinitum

M.C. Escher,
Moebius Ants

CS655: Programming Languages
University of Virginia
Computer Science

David Evans
http://www.cs.virginia.edu/~evans

Goal: Understand the Least Fixed Point Theorem

If D is a ***pointed complete*** partial order, then a ***continuous function*** $f\colon D \to D$ has a least fixed point ($\mathbf{fix}_D\ f$) defined by

$$\bigsqcup_D \{\ (f^n\ \bot_D) \mid n \geq 0\ \}$$

Last Time

- A *domain* is a structured set of values
- A *function domain* is constructed from two primitive domains, $D_1 \rightarrow D_2$ by associating an element of D_2 with each element of D_1.
- A *fixed point* of a function f: $D_1 \rightarrow D_2$ is an element $d \in D$ such that $f\, d = d$.