Extra Counting Practice Solutions

1. How many 6-digit numbers are possible if the first two digits cannot be 0, and there can be no repeated digits?

   1st digit: Cannot be 0 – 9 choices
   2nd digit: Cannot be 0 or the first digit – 8 choices
   3rd digit: Cannot be first or second digit – 8 choices
   4th digit: Cannot be first, second, or third digit – 7 choices
   5th digit: Cannot be first, second, third, or fourth digit – 6 choices
   6th digit: Cannot be first, second, third, fourth, or fifth digit – 5 choices

   $$\underline{9} \cdot \underline{8} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} = \boxed{120{,}960}$$

2. How many 5-digit numbers are possible if the first digit must be odd, the second digit cannot be 0, and the first 3 digits must be different?

   1st digit: Must be odd – 5 choices
   2nd digit: Cannot be 0 or the first digit – 8 choices
   3rd digit: Cannot be first or second digit – 8 choices
   4th digit: No restrictions – 10 choices
   5th digit: No restrictions – 10 choices

   $$\underline{5} \cdot \underline{8} \cdot \underline{8} \cdot \underline{10} \cdot \underline{10} = \boxed{32{,}000}$$

3. How many 6-digit numbers are possible if the last digit must be even but not 0, the first digit cannot be 0 or 1 and there can be no repeated digits?

   Start with choosing last digit, since it has the most restrictions.
   6th (last) digit: Must be even, not 0 – 4 choices
   1st digit: Cannot be 0, 1, or last digit – 7 choices
   2nd digit: Cannot be first or last digit – 8 choices
   3rd digit: Cannot be first, second, or last digit – 7 choices
   4th digit: Cannot be first, second, third, or last digit – 6 choices
   5th digit: Cannot be first, second, third, fourth, or last digit – 5 choices

   $$\underline{7} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} = \boxed{47{,}040}$$

4. How many 5-letter "words" are possible if the word must start and end with a vowel and the remaining letters must be different consonants?

   1st letter: Must be a vowel – 5 choices
   5th (last) letter: Must be a vowel – 5 choices
   2nd letter: Must be a consonant (not a vowel) – 21 choices
   3rd letter: Must be a consonant, cannot be 2nd letter – 20 choices
   4th letter: Must be a consonant, cannot be 2nd or 3rd letter – 19 choices

   $$\underline{5} \cdot \underline{21} \cdot \underline{20} \cdot \underline{19} \cdot \underline{5} = \boxed{199{,}500}$$

5. A jar contains 21 jelly beans: 4 red, 5 green, 7 white, 3 black, and 2 orange. A sample of 6 jelly beans is chosen.

   (a) How many samples are possible?

   $$C(21,6) = \boxed{54{,}264}$$

(b) How many samples contain no whites?
Sample contains: 0 white, 6 not white

$$C(7,0)C(14,6) = C(14,6) = \boxed{3{,}003}$$

(c) How many samples contain exactly 2 black?
Sample contains 2 black, 4 not black

$$C(3,2)C(18,4) = \boxed{9{,}180}$$

(d) How many samples contain exactly 4 white and exactly 2 red?

$$C(7,4)C(4,2) = \boxed{210}$$

(e) How many samples contain exactly 3 green and exactly 2 black?
Sample contains 3 green, 2 black, 1 not green or black

$$C(5,3)C(3,2)C(13,1) = \boxed{390}$$

(f) How many samples contain exactly 3 red or exactly 5 white?
Two cases combined using union rule:
n(ex 3 R or ex 5 W)= n(ex 3 R)+n(ex 5 W)−n(ex 3 R and ex 5 W)

n(ex 3 R and ex 5 W)= 0 since it is not possible to have 8 jelly beans in a sample of 6.

3 red, 3 not red   5 white, 1 not white

$$C(4,3)C(17,3) + C(7,5)C(14,1) = \boxed{3{,}014}$$

(g) How many samples contain exactly 2 black or exactly 4 green?
Two cases combined using union rule:
n(ex 2 B or ex 4 G)= n(ex 2 B)+n(ex 4 G)−n(ex 2 B and ex 4 G)

2 B, 4 not B   4 G, 2 not G   2 B, 4 G

$$C(3,2)C(18,4) + C(5,4)C(16,2) - C(3,2)C(5,4) = \boxed{9{,}765}$$

(h) How many samples contain at least 1 white?

Total number of samples − Number of samples with no white

$$C(21,6) - C(14,6) = \boxed{51{,}261}$$

(i) How many samples contain at least 3 red?
At least 3 red means exactly 3 red OR exactly 4 red. (There can't be more than 4 red.)

$$C(4,3)C(17,3) + C(4,4)C(17,2) = \boxed{2{,}856}$$

(j) How many samples contain at most 3 green?
At most 3 green means 0, 1, 2, OR 3 green.
One method is to add up these four cases.

$$C(5,0)C(16,6) + C(5,1)C(16,5) + C(5,2)C(16,4) + C(5,3)C(16,3) = \boxed{53{,}648}$$

Another method is to take the total number of samples and subtract the number of samples you don't want (the complement). The samples you don't want are those with 4 or 5 green.

$$C(21,6) - [C(5,4)C(16,2) + C(5,5)C(16,1)] = \boxed{53{,}648}$$

6. John has 12 CD's in his car: 3 are country, 3 are rap, 4 are classical, and 2 are blues.

   (a) In how many ways can all 12 CDs be arranged in a CD holder?

   $$12! = \boxed{479{,}001{,}600}$$

   (b) In how many ways can 9 of 12 CDs be arranged in a CD holder?

   $$P(12,9) = \underline{\ 12\ } \cdot \underline{\ 11\ } \cdot \ldots \cdot \underline{\ 4\ } = \boxed{79{,}833{,}600}$$

   (c) In how many ways can all 12 CD's be arranged in a CD holder if each type of music is put together?

   Arrange the four groups (types) of music: 4!
   Arrange CDs in country group: 3!
   Arrange CDs in rap group: 3!
   Arrange CDs in classical group: 4!
   Arrange CDs in blues group: 2!

   $$\underline{\ 4!\ } \cdot \underline{\ 3!\ } \cdot \underline{\ 3!\ } \cdot \underline{\ 4!\ } \cdot \underline{\ 2!\ } = \boxed{41{,}472}$$

7. In how many distinguishable ways can the letters in the word SENSELESSNESS be arranged?

   Total number of letters: 13
   Number of identical S's: 6
   Number of identical E's: 4
   Number of identical N's: 2
   Number of identical L's: 1

   $$\frac{13!}{6! \cdot 4! \cdot 2!} = \boxed{180{,}180}$$

8. A boss is interviewing candidates in order to hire a manager, 2 computer programmers, 3 program coordinators, and 4 secretaries. The secretaries will be chosen from a pool of 9 candidates and the remaining positions will be chosen from among 8 qualified candidates. In how many ways can the boss fill these positions?

   Number of ways to hire manager: $C(8,1) = 8$
   Number of ways to hire 2 computer programmers: $C(7,2)$
   Number of ways to hire 3 program coordinators: $C(5,3)$
   Number of ways to hire 4 secretaries: $C(9,4)$

   $$8 \cdot C(7,2) \cdot C(5,3) \cdot C(9,4) = \boxed{211{,}680}$$