Claim Let $N(i,j)$ denote the # of nodes in recursion tree (i.e., # of rec. calls). Then $N(i,j) = 2\binom{i+j}{j} - 1$

Pf. Observe $N(i,j) = N(i-1,j) + N(i,j-1) + 1$

Thus, by IH, we have:

$$N(i,j) = 2\binom{i+j-1}{j} - 1 + 2\binom{i+j-1}{j-1} - 1 \quad + 1$$
$$= 2\left[\binom{i+j-1}{j} + \binom{i+j-1}{j-1}\right] - 1$$

Noting that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$,

$$N(i,j) = 2\binom{i+j}{j} - 1 \qquad \boxtimes$$

Claim. $\binom{2n}{n} \geq \frac{4^n}{2n+1}$

Pf. $\binom{2(n+1)}{n+1} = \binom{2n+2}{n+1} = \binom{2n+1}{n+1} + \binom{2n+1}{n}$

$$= 2\binom{2n+1}{n+1} = 2\left[\binom{2n}{n+1} + \binom{2n}{n}\right]$$

$$= 2\left[\binom{2n}{n}\frac{n}{n+1} + \binom{2n}{n}\right] = 2\binom{2n}{n}\frac{2n+1}{n+1}$$

IH $\Rightarrow$ $\geq 2\,\frac{4^n}{2n+1} \cdot \frac{2n+1}{n+1} = 4^n \cdot \frac{2}{n+1}$

$$\geq 4^n \cdot \frac{4}{2n+3} = \frac{4^{n+1}}{2(n+1)+1} \qquad \boxtimes$$