5/20

$$q'' = -\left(\frac{dT}{dx}\vec{i}\right)k$$

Chapter 2

thermal diffucitivity

2-19 $\rho C_p \frac{dT}{dt}$

2-21 $\frac{1}{\alpha}\frac{dT}{dt}$

$$\alpha = \frac{k}{\rho C_p} \quad \frac{\left[\frac{W}{mK}\right]}{\left[\frac{kg}{m^3}\right]\left[\frac{J}{kg\,K}\right]} \rightarrow \frac{Wm^2}{J} \rightarrow \frac{m^2}{s}$$

$$\frac{k}{\rho C_p} = \alpha$$

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen}$$

$$mc\frac{dT}{dt} = q''A\big|_x - q''A\big|_{x+\Delta x} + \dot{q}(A\Delta x)$$

$$\frac{\rho(A_{cs}\Delta x)c\frac{dT}{dt}}{A\Delta x} = \frac{-A\left(k\frac{dT}{dx}\Big|_x\right)}{A\Delta x} + \frac{A\left(k\frac{dT}{dx}\Big|_{x+\Delta x}\right)}{A\quad\Delta x} + \dot{q}\frac{A}{A}\frac{\Delta x}{\Delta x}$$

$$f(x) = \left(k\frac{dT}{dx}\right)$$

$$\rho c\frac{dT}{dt} = \frac{\left(k\frac{dT}{dx}\right)\Big|_{x+\Delta x} - \left(k\frac{dT}{dx}\right)\Big|_x}{\Delta x} + \dot{q}$$

$$\rho c\frac{dT}{dt} = \frac{f(x+\Delta x) - f(x)}{\Delta x} + \dot{q}$$

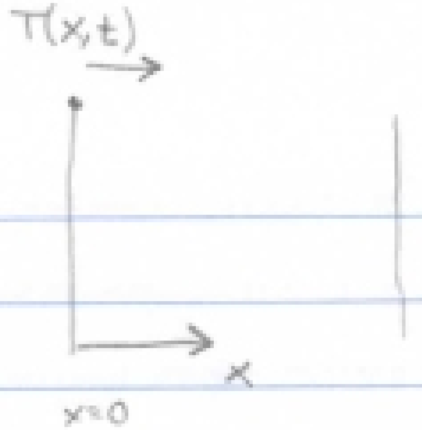
$$\rho c\frac{dT}{dt} = \frac{df}{dx} + \dot{q} \longrightarrow \rho c\frac{dT}{dt} = \frac{\partial}{\partial x}\left(k\frac{dT}{dx}\right) + \dot{q} \quad (2.19)$$

2-19 $\frac{\partial}{\partial x}\left(k\frac{dT}{dx}\right) + \dot{q} = \rho c\frac{dT}{dt}$

$$\boxed{\text{2-21} \quad \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{dT}{dt}} \quad T(x,t)$$

Boundary Conditions

5/21 Table 2.2 Boundary Conditions



left

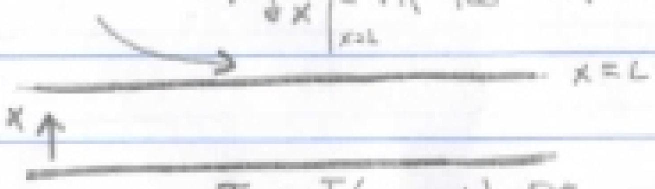
1.) $T(x=0,t) = T_L$

2.) (a) $q''_L = -k \frac{\partial T}{\partial x}\Big|_{x=0}$

(b) $0 = \frac{\partial T}{\partial x}$

3.) $-k\frac{\partial T}{\partial x}\Big|_{x=0} = h\left[T_\infty - T(0,t)\right]$

$k\frac{\partial T}{\partial x}\Big|_{x=L} = h(T_\infty - T(x=L,t))$ BC_R



IC

$T_0 = T(x,t=0)$

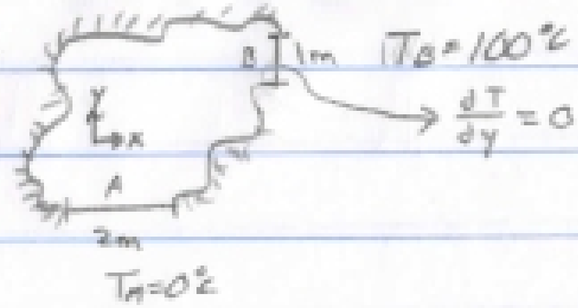
$T_0 = T(x=0,t)$ BC_L

Example 2.4

(2.21) $\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

Chapter 2 Homework - 2.15

pop quiz on tues



$q = 10$ watts

$q''_B = 10 \frac{W}{m^2}$

$q''_A = 5 \frac{W}{m^2}$

$\frac{\partial T}{\partial y} = \frac{30\,k}{m}$

$\vec{q}'' = -k\left(\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \cancel{\frac{\partial T}{\partial z}}^{\,0}\right)$

$\vec{q}''_B = -k\,\frac{\partial T}{\partial x}\Big|_B \vec{i}$

$\vec{q}''_A = -k\frac{\partial T}{\partial y}\Big|_A \vec{j} = \frac{1}{2}q''_B = \frac{1}{2}\left(-k\frac{\partial T}{\partial x}\Big|_B\right)$

$\hookrightarrow \cancel{k}\frac{\partial T}{\partial y}\Big|_A = \frac{1}{2}\cancel{k}\frac{\partial T}{\partial x}\Big|_B$

$\hookrightarrow \frac{\partial T}{\partial y}\Big|_A = \frac{1}{2}\frac{\partial T}{\partial x}\Big|_B$ $\qquad \frac{\partial T}{\partial y}\Big|_A = \frac{1}{2}\,\frac{30\,k}{m}$

1.) $\frac{d^2T}{dx^2} = \frac{hP}{Ak}\underbrace{(T - T_\infty)}_{z}$ $T(0) = 100°c$, $\lim_{x\to\infty} T = T_\infty$, $\frac{hP}{Ak} = 4.0 m^{-2}$, $T_\infty = 30°c$

$\frac{d^2T}{dx^2} = \frac{hP}{Ak}(z)$

5 $\frac{d^2y}{dx^2} = C_1 y \rightarrow y(x) = C_2 e^{-\sqrt{C_1}x} + C_3 e^{\sqrt{C_1}x}$

$C_1 = \frac{hP}{Ak} = 4\frac{1}{m^2}$

$y = z$

$z(x) = C_2 e^{-\sqrt{4\frac{1}{m^2}}x} + C_3 e^{\sqrt{4\frac{1}{m^2}}x}$

when $\lim_{x\to\infty} T = T_\infty \rightarrow \lim_{x\to\infty} z = 0$

$\hookrightarrow = C_2 e^{-2(\infty)} + \cancel{C_3} e^{2(\infty)} = 0 \longrightarrow C_3 = 0$

$\rightarrow T - T_\infty = C_2 e^{-2x}$

$100 - 25 = C_2 e^{-2(0)} \longrightarrow C_2 = 75°c$

$\boxed{T = 75°c\, e^{-2\frac{x}{m}} + 25°c}$

2.) $\frac{dT}{dt} = \frac{hA}{mc}(T_\infty - T)$ $T(0) = 10°c$, $T_\infty = 30°c$, $\frac{hA}{mc} = 2.0\ s^{-1}$

$\frac{dT}{dt} = -\frac{hA}{mc}T + \frac{hA}{mc}T_\infty \rightarrow \frac{dT}{dt} = -2.0\frac{1}{s}T + 2.0\frac{1}{s}(30°c)$

$\frac{dT}{dt} = -2\frac{1}{s}T + 60\frac{°c}{s}$ $z = y - \frac{C_2}{C_1} \rightarrow z = T - 30$

$C_1 = \frac{hA}{mc} = 2\frac{1}{s}$

$C_2 = \frac{hA}{mc}(T_\infty) = 60\frac{°c}{s}$

$z_0 = T_0 - \frac{60°c}{2} \longrightarrow z_0 = 10°c - 30°c$ $z_0 = -20°c$

$z = z_0 e^{-\frac{hA}{mc}(t)} \longrightarrow T - 30°c = -20°c\, e^{-2t/s}$

$T(t) = 30°c - 20°c\, e^{-2t/s}$