

## Practice Final Exam: Winter 2007

1. (40 points) Find  $\frac{dy}{dx}$  for each function. Each answer should be a function of  $x$  only.

(a) (10 points)  $y = \frac{2}{x-1} - \frac{1}{\sqrt{x}}$ .

(b) (10 points)  $y = (\sin x)^{\cos x}$ .

(c) (10 points)  $y = \sqrt{\tan(x^2)}$ .

(d) (10 points)  $y = \frac{(2x+1)^4 \sin(x^2)}{(\ln x)\sqrt{3x-1}}$ .

2. (10 points) Find the equation of the tangent line to the curve

$$e^{x^2} + e^{y^2} = 2e$$

at the point  $(-1, 1)$ .

3. (20 points) Let

$$f(x) = \ln(x^2 - 1).$$

(a) (10 points) You must show all your work, but please write your final answers in the box.

The domain of $f(x)$ is:	
$f(x)$ is increasing on:	
$f(x)$ is decreasing on:	
$f(x)$ has local maxima at:	
$f(x)$ has local minima at:	
$f(x)$ is concave up on:	
$f(x)$ is concave down on:	

(b) (4 points) Compute the following four limits.

$$\lim_{x \rightarrow \infty} \ln(x^2 - 1) =$$

$$\lim_{x \rightarrow -\infty} \ln(x^2 - 1) =$$

$$\lim_{x \rightarrow 1^+} \ln(x^2 - 1) =$$

$$\lim_{x \rightarrow -1^-} \ln(x^2 - 1) =$$

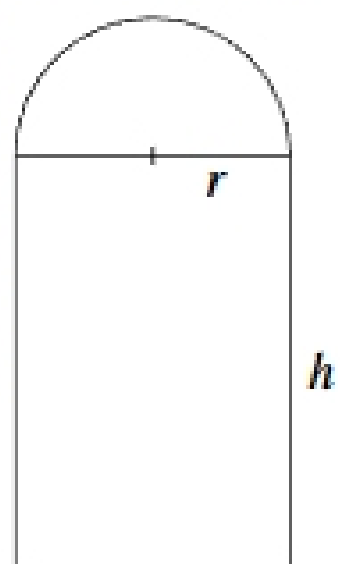
- (c) (1 points) List all vertical and horizontal asymptotes of  $y = \ln(x^2 - 1)$ .
- (d) (5 points) Using your answers from parts (a) and (b), sketch a graph of

$$f(x) = \ln(x^2 - 1).$$

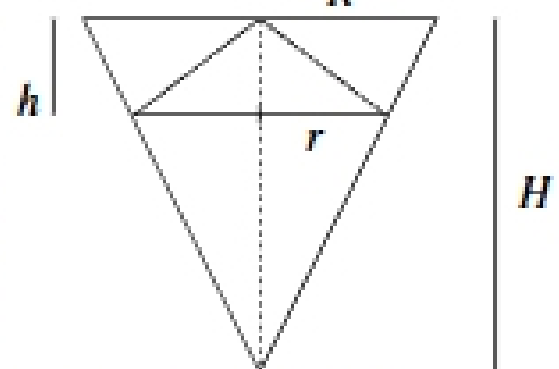
Even if your answers in parts (a) and (b) are wrong, if your sketch correctly uses those answers, you may earn partial credit.

4. (20 points) A particle is moving along the curve  $x^2 - 4xy - y^2 = -11$ . Given that the  $x$ -coordinate of the particle is changing at 3 units/second, how fast is the distance from the particle to the origin changing when the particle is at the point (1,2)? Hint: As an intermediate step, you should compute the value of  $\frac{dy}{dt}$  when  $x = 1$  and  $y = 2$ .
5. (20 points) A balloon is rising at a constant speed of 1 m/sec. A girl is cycling along a straight road at a speed of 2 m/sec. When she passes under the balloon it is 3 m above her. How fast is the distance between the girl and the balloon increasing 2 seconds later?

6. (20 points) A Norman window consists of a rectangle surmounted by a semicircle, as shown. Given that the total area of the window is  $A = 8 + 2\pi$ , find the minimum possible perimeter of the window. (Please note the horizontal line between the rectangle and the semicircle does not count as part of the perimeter.) Hint: The total area has been carefully chosen so that the minimum perimeter occurs at a very simple value of  $r$ . If your optimal value of  $r$  is complicated, you have done something incorrectly.



7. (20 points) Suppose you have a cone with constant height  $H$  and constant radius  $R$ , and you want to put a smaller cone "upside down" inside the larger cone (see figure). If  $h$  is the height of the smaller cone, what should  $h$  be to maximize the volume of the smaller cone? The optimal value of  $h$  will depend on  $H$ . Recall that the volume of a cone with base radius  $r$  and height  $h$  is given by the formula  $V = \frac{1}{3} \pi r^2 h$ .



8. (10 points) For parts (a) and (b), compute the given limits, if they exist. If you assert that a limit does not exist, you need to justify your answer to get full credit.

(a) (5 points)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 1} - \sqrt{x^2 + 2})$

(b) (5 points)  $\lim_{x \rightarrow 2} e^{\frac{1}{x-2}}$