Combined Bending and Axial Loads

Related Material

AISC LRFD Manual: Part 6
AISC LRFD Specifications: Chapter C, Chapter H

Two New Concepts

Superposition of stresses due to bending and axial loads
Secondary moment due to axial loads; moment amplification

Interaction Equations

- Strength interaction equations relating axial compression P_u to bending moment M_u have been recognized as the practical procedure for design.

- Axial compression strength requirement

  Required axial strength ≤ Design axial strength of the section

  $$P_u \le \phi_c P_n, \text{ or } \frac{P_u}{\phi_c P_n} \le 1$$

- Bending moment strength requirement

  Required bending strength ≤ Design bending strength of the section

  $$M_u \le \phi_b M_n, \text{ or } \frac{M_u}{\phi_b M_n} \le 1$$

- Combined bending and axial compression: Interaction equation

  $$\frac{P_u}{\phi_c P_n} + \frac{M_u}{\phi_b M_n} \le 1$$

LRFD Criteria

- For $\dfrac{P_u}{\phi_c P_n} \geq 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_u}{\phi_b M_n}\right) \leq 1$$

- For $\dfrac{P_u}{\phi_c P_n} < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_u}{\phi_b M_n}\right) \leq 1$$

$$\phi_c = 0.85,\ \phi_b = 0.90$$

P_u = Factored axial compression load

M_u = Factored bending moment (moment magnification used)

$\dfrac{P_u}{\phi_c P_n} \geq 0.2$, large axial load, bending term is slightly reduced.

$\dfrac{P_u}{\phi_c P_n} < 0.2$, small axial load, axial load term is reduced.

P_n = Nominal axial strength of the section

M_n = Nominal bending strength of the section

Moment Amplification

- *Beam-column*: the member subjected to axial compression and bending. Axial load induces additional moment, called secondary moment that must be accounted for in design. Problem is nonlinear, requiring second order analysis. AISC permits use of *moment amplification method* or second order analysis.

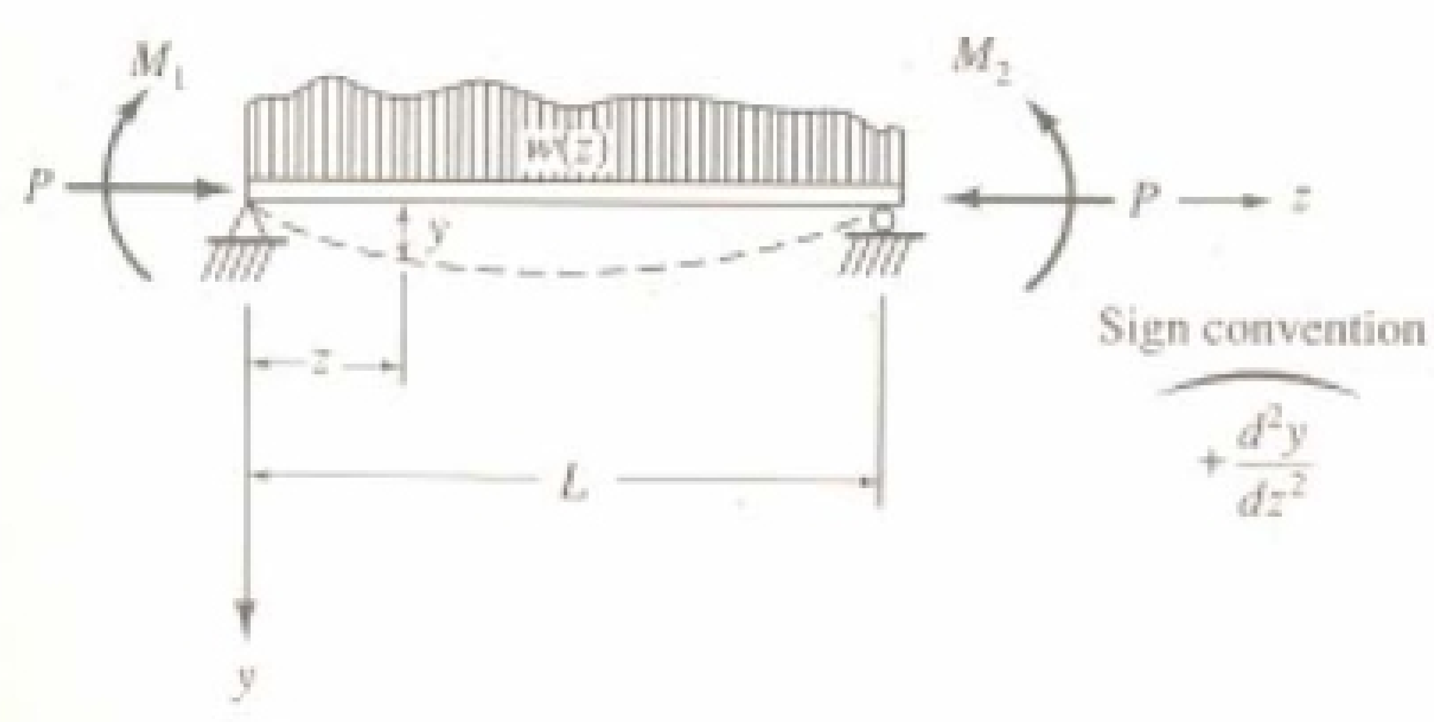

Figure 12.2.1 General loading of beam-column.

- Maximum moment: Note that the coordinate system here is different from the one used earlier.

$$M_z = M_i + Py = -EI\frac{d^2 y}{dz^2}$$

Solution:

-- (1) If $w(z) = 0$ and $M_1 = M_2 = M$,

$$M_{z\max} = M\sec\frac{kL}{2}, \text{ where } k = \sqrt{P / EI}$$

-- (2) If $w(z) = w$ and $M_1 = M_2 = 0$,

$$M_{z\,max} = \frac{wL^2}{8}\left(\frac{8}{(kL)^2}\right)\left(sec\frac{kL}{2} - 1\right)$$

It can be shown that

$$M_{max} = M_0\left(\frac{1}{1-\alpha}\right); \;\; \alpha = \frac{P_u}{P_e}$$