

Lecture Ch. 4b

- Hydrostatic equilibrium
 - Special cases
 - Pressure altitude dependence
- More Midterm Review problems
 - Terminology review

Curry and Webster, Ch. 4 (pp. 96-115; skip 4.5 (except 4.5.1), 4.6)
 Tuesday, Oct. 20: **Homework**, Review (bring questions), and Read Ch. 4
 Tuesday, Oct. 27: **Midterm**
 Thursday, Oct. 29: meet to work on ROAST!

Water Vapor Metrics

The water vapor mixing ratio, w , is the ratio of the mass of water vapor present to the mass of dry air. It is thus defined, after subtracting from the ideal gas law, as

$$w = \frac{m_w}{m_d} = \frac{p_w}{p_d} = \frac{e}{p - e} \quad (1.35)$$

where m_w/m_d is 6.22 (Section 1.7). A ratio of the mass of water vapor to the mass of dry air is

$$w = e_p \frac{p_0}{p} \quad (1.37)$$

Using $p = p_0 - \rho g z$,

$$w = \frac{e_p}{p_0} \left(1 + \frac{\rho g z}{p_0} \right) \quad (1.38)$$

is an approximate definition of the relative humidity.

The water vapor mixing ratio can be related to the specific humidity, q , which was originally defined in Section 1.7 as

$$q = \frac{m_w}{m_w + m_d} = \frac{e}{p} = \frac{w}{1 + w} \quad (1.39)$$

Equivalently, $w = q$ or $q = w/(1 + w)$.

Special Cases of Hydrostatic Equilibrium

- $\rho = \text{constant}$ (homogeneous)
 - $H = 8 \text{ km} = RT/g = \text{scale height}$ eq. 1.39
- constant lapse rate (implied if hydrostatic, homogeneous, and ideal gas)
 - $-dT/dz = \text{constant} = g/R = -34 \text{ deg/km}$
- isothermal $T = \text{constant}$ (and ideal gas)
 - $p = p_0 \exp(-z/H)$

Special Cases of Hydrostatic Equilibrium

- Hydrostatic: Force balance on gravity and upward pressure

... is a vertical acceleration in the direction of decreasing pressure (upwards). The vertical pressure gradient force is generally in very close balance with the downward force due to gravitational attraction. This is called *hydrostatic balance*, and is written as

$$g = -\frac{1}{\rho} \frac{dp}{dz} \quad (1.33)$$

where g is the acceleration due to the Earth's gravity. The hydrostatic balance is applicable to most situations in the atmosphere and ocean, exceptions arising in the presence of large vertical accelerations such as are associated with thunderstorms.

Homogeneous Atmosphere

Homogeneous
 Constant density
 Constant lapse rate

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$$g = -\frac{1}{\rho} \frac{dp}{dz} \quad (1.33)$$

The height of the homogeneous atmosphere is defined as the scale height H .

$$H = \frac{RT}{g} = \frac{p}{\rho g} \quad (1.39)$$

Using $H = 8 \text{ km}$ and $T = 288 \text{ K}$, we find $H = 8.4 \text{ km}$. This is the scale height of a homogeneous atmosphere. The height of the homogeneous atmosphere is defined as the scale height H .

$$H = \frac{RT}{g}$$

Combining 1.33 with the hydrostatic equation (1.33) leads to the result

$$p = p_0 \exp\left(-\frac{z}{H}\right) = p_0 \exp\left(-\frac{gz}{RT}\right)$$

The height scale for the homogeneous atmosphere is referred to as the *scale height*. It is a measure of the vertical distance over which the pressure changes by a factor of e . The scale height is a function of the temperature and the mean molecular weight of the atmosphere. The scale height is a function of the temperature and the mean molecular weight of the atmosphere. The scale height is a function of the temperature and the mean molecular weight of the atmosphere.

Isothermal Atmosphere

Further, the p is related to the vertical distance z by the hydrostatic equation. After substituting the ideal gas law for density, we can write the hydrostatic equation in the following form

$$dp = -\frac{p}{H} dz \quad (1.40)$$

This equation is easily integrated for a constant temperature from sea level ($z = 0$, $p = p_0$) to some arbitrary height z :

$$\int_{p_0}^p \frac{dp}{p} = -\frac{1}{H} \int_0^z dz \quad (1.41)$$

or

$$\ln \frac{p}{p_0} = -\frac{z}{H} \quad (1.42)$$

Taking exponents and using $H = RT/g$, we have

$$p = p_0 \exp\left(-\frac{gz}{RT}\right)$$

This equation describes isothermal atmosphere. The vertical pressure gradient force is generally in very close balance with the downward force due to gravitational attraction. This is called *hydrostatic balance*, and is written as

Hydrostatic Equilibrium Example

Consider a planet with an atmosphere in hydrostatic equilibrium. Assume that the atmosphere is an ideal gas. Also assume that the temperature is a maximum at the surface of the planet, and, as height increases, the temperature in the atmosphere decreases linearly (in other words, temperature decreases with height at a constant rate). Derive a formula for atmospheric density as a function of height in this atmosphere.

Quiz

Answer briefly and clearly, with appropriate equations or diagrams.

- If a homogeneous mixture of two substances coexists in liquid and vapor phases at a series of pressures (each of which corresponds to exactly one temperature), how many degrees of freedom are in this system?
- What is thermal equilibrium? Give an equation.
- Give the equation for the Gibbs phase rule.
- What is the change in free energy for a phase change?
- What type of pressure change is described by the Clausius-Clapeyron equation? i.e. what changes as a function of what under what conditions?

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Answers

- + If a homogeneous mixture of two substances coexists in liquid and vapor phases at a series of pressures (each of which corresponds to exactly one temperature), how many degrees of freedom are in this system?
 - 1
- + What is thermal equilibrium? Give an equation.
 - $T_1 = T_2$
- + Give the equation for the Gibbs phase rule.
 - $f = c - p + 2$
- + What is the change in free energy for a phase change?
 - 0 (at constant T and P)
- + What type of pressure change is described by the Clausius-Clapeyron equation? i.e. what changes as a function of what under what conditions?
 - $p_2 = f(T)$ for 1-V transition pressure

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Hydrostatic Equilibrium Example

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$$p = \rho g h$$

$$\text{and a constant temperature } T = T_0 - \alpha h$$

$$\rho = \frac{p}{RT} \left(\frac{1}{T_0 - \alpha h} \right) = \frac{p}{RT_0} \left(\frac{T_0}{T_0 - \alpha h} \right)$$

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Clausius Clapeyron Example

The Clausius Clapeyron equation is a function of T and p . The Clausius Clapeyron equation is $\frac{dp}{dT} = \frac{\Delta H_{vap}}{T^2} \left(\frac{1}{V_g} - \frac{1}{V_l} \right)$.

In addition to the Clausius Clapeyron equation, we also have the ideal gas law $pV = nRT$. Assuming the volume of the liquid is negligible compared to the volume of the gas, we can simplify the Clausius Clapeyron equation to $\frac{dp}{dT} = \frac{\Delta H_{vap}}{T^2} \left(\frac{1}{V_g} \right)$.

The Clausius Clapeyron equation can be integrated if ΔH_{vap} is assumed constant, and the result is eq. 11.11. Using this, we can derive the Clausius Clapeyron equation for a phase transition between two states, as any volume law. See eq. 11.11.

$$\ln \left(\frac{p_2}{p_1} \right) = \frac{\Delta H_{vap}}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

$$\ln \left(\frac{p_2}{p_1} \right) = \frac{\Delta H_{vap}}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right) = \frac{\Delta H_{vap}}{R} \left(\frac{T_2 - T_1}{T_1 T_2} \right) = \frac{\Delta H_{vap}}{R} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$$

See eq. 11.11.

Degrees of Freedom Example

How many degrees of freedom are there in the system? (a) if the system is a gas, (b) if the system is a liquid, (c) if the system is a solid, (d) if the system is a mixture of gas and liquid, (e) if the system is a mixture of gas and solid, (f) if the system is a mixture of liquid and solid, (g) if the system is a mixture of gas, liquid, and solid.

See eq. 11.11. Using this, we can derive the Clausius Clapeyron equation for a phase transition between two states, as any volume law. See eq. 11.11.

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Quiz

Answer briefly and clearly, with appropriate equations or diagrams.

- If a pure substance coexists in liquid and vapor phases at a series of pressures (each of which corresponds to exactly one temperature), how many degrees of freedom are in this system?
- What is the relationship between two pressures at mechanical equilibrium?
- Give the equation for the Gibbs phase rule.
- What three types of equilibrium are required by the Gibbs phase rule?

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