

COSC 4368: Fundamentals of Artificial Intelligence Spring 2019
Problem Set3 (Individual Tasks) Version 4

Deadline: Su. April 28, **11p** (3% bonus); Wednesday, May 1, **11a** (the latest)
Available Points: 57

12. Logical Reasoning (4 points) Khadija

Show using Resolution (and **not** by using other methods!):

- (1) $\forall x \forall y \exists z (P(x,y,z) \rightarrow R(x,y))$
- (2) $\forall r \forall s (P(s,s,t) \rightarrow Q(s,t))$
- (3) $\forall a \exists b (Q(a,a) \rightarrow R(b,a))$
- (4) $\forall x \forall y (R(x,y) \rightarrow R(y,y))$
- (5) $P(4,4,4)$
- (6) $\neg Q(4,5)$
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- (X) $R(4,4)$

First transform the FOPL formulas into clauses, and then the hunt for the empty clause can begin!

13. Bayes' Theorem and Belief Networks (9 points) Romita



Fig. 1: Thomas Bayes \approx 1740—279 years ago!

- a) Assume we have 3 symptoms S_1, S_2, S_3 , a disease D and the following probabilities: $P(D)=0.01$ $P(S_1)=P(S_2)=P(S_3)=0.02$; $P(S_1|D)=0.1$; $P(S_2|D)=0.02$; $P(S_3|D)=0.002$. How would a naïve Bayesian system compute the following probability? $P(D|S_1,S_2,S_3)=\dots$
- b) Now assume the following additional knowledge has become available: $P(S_1,S_2)=0.0002$; $P(S_3|S_1,S_2)=0.08$; $P(S_1,S_2,S_3|D)=0.000032$; how would you use this information to obtain a “better” estimation of $P(D|S_1,S_2,S_3)$?
- c) How can the discrepancy with respect to the obtained probabilities between cases a) and b) be explained? Why are the numbers you obtain different? What does this discrepancy tell you about naïve Bayesian systems in general?

15. Using a Belief Network Tool (20 points) Khadija



Fig. 3: Multiple Astronomers Looking at the Sky

Assume we have 3 astronomers in different parts of the world who make measurements M_1 , M_2 , and M_3 of the number¹ of stars N in some region of the sky. Normally, there is a probability of 0.05 that the astronomer counts a single star twice (overcounts by one star; you can assume that the three astronomers never undercount; moreover, if there is no star visible ($N=0$) the astronomer never overcounts). Moreover, there is a 10% probability ($P(F_i=1)=0.1$ for $i=1,2,3$) that a telescope is out of focus (represented using random variables F_1 , F_2 , and F_3), in which the astronomer undercounts by 2 or more stars (e.g. if N is 4 and her telescope is out of focus, the astronomer will count 2, 1 or 0 stars; you can assume if information is missing that each case has the same probability). Design a belief network, and compute the probability of the other variables assuming the following pieces of evidence are given (feel free to use *Netica* (<http://www.norsys.com/download.html>) or any another belief network tool to compute your answer²):

1. $M_1=3$ $M_2=3$ $M_3=1$
2. $M_1=3$ $M_2=3$ $M_3=0$
3. $N=4$, $M_2=1$, $M_3=0$
4. $M_1=0$ $M_2=3$ $M_3=2$
5. $N=3$ $F_1=0$ $F_2=0$ $F_3=1$
6. $M_1=4$ $M_2=4$ $F_3=1$

Submit the complete Belief Network you created—including all its probability tables—and the findings you obtained for the six cases listed above!

¹ You can assume that N is limited to 4—but the astronomer do not know that: M_1 , M_2 and M_3 are therefore limited to values 0 through 5.

² Including the answer 'inconsistent' in the case that the evidence is inconsistent, e.g. the evidence $N=1$ $M_1=3$ is inconsistent—as it is 'impossible' because astronomer1 never overcounts by more than 1 star!