

## Lesson 38 MA 15200, Appendix I Section 5.5

You are familiar with the simple interest formula,  $I = prt$ . However, in many accounts the interest is left in the account and earns interest also. We say the account earns **compound interest**.

For example: Suppose Bob invests \$100 at 10% simple interest. At the end of 1 year, Bob has earned  $I = 100(.10)(1) = \$10$ . He now has \$110.

At the end of the 2<sup>nd</sup> year, Bob has earned  $I = 110(.10)(1) = \$11$ . He now has \$121.

At the end of the 3<sup>rd</sup> year, Bob has earned  $I = 121(.10)(1) = \$12.10$ . He has a total of \$143.10. I'm sure you get the idea of what is happening.

**Formula for Compound Interest with Annual compound interest:**

$S = P(1+r)^t$ , where  $P$  is the initial investment (principal),  
 $t$  is the number of years,  $r$  is the annual interest rate, and  $S$   
is the future value or final value.

Ex 1: Assume that \$1500 is deposited in an account in which interest is compounded annually at a rate of 6%. Find the accumulated amount after 5 years.

Ex 2: Assume that \$1500 is deposited in an account in which interest is compounded annually for 5 years. Find the accumulated amount, if the interest rate is  $8\frac{1}{2}\%$ .

Many banks or financial institutions figure interest more often than once a year; quarterly monthly, semiannually, daily, etc. For example, if the annual rate or **nominal rate** is 12% and interest is compounded quarterly, that is equivalent to 3% every 3 months. 3% is called the **periodic rate**.

**Formula for Periodic Rate:** Periodic Rate =  $\frac{\text{annual rate}}{\text{number of periods per year}}$

$i = \frac{r}{k}$ , where  $r$  is annual interest rate,  $k$  is the number of times

interest is paid each year, and  $i$  is the periodic rate.

**Ex 3:** Find the periodic rate in each example.

- a) annual rate: 10%, compounded quarterly
- b) annual rate: 3.6%, compounded monthly

**Compound Interest Formula (Future Value of an Investment):**

Let  $P$  be principal earning interest compounded  $k$  times per year for  $n$  years at an annual rate of  $r$ . Then, the final or future value will be

$$* S = P(1+i)^{kt}, \text{ where } i = \frac{r}{k}$$

\*Earlier in the semester, when we had this formula, it was written  $A = P\left(1 + \frac{r}{n}\right)^{nt}$ , where

$A$  is the final amount,  $P$  is principal or beginning amount,  $r$  is annual interest rate,  $n$  is number of compounding periods a year, and  $t$  is time in years. This lesson the formula is simply written differently.

**Ex 4:** Assume that \$1500 is deposited in an account in which interest is compounded monthly at an annual rate of 6%.

- a) Find the accumulated amount after 8 years.
- b) How much interest was earned during the 8 years?

Financial institutions are required to provide customers with the **effective rate of interest**, that rate at which, if compounded annually, would provide the same yield as the plan where interest is compounded more frequently.

In other words: For what interest rate is  $P(1+r)^n = P(1+i)^{kn}$ ? If this equation is solved for  $r$ , we get the following formula.

**Effective Rate of Interest:** The effective rate of interest  $R$  for an account paying a nominal or annual interest rate  $r$ , compounded  $k$  times per year is....

$$E = (1+i)^k - 1, \text{ where } i \text{ (the periodic rate)} = \frac{r}{k}.$$

Ex 5: Find the effective rate of interest given the annual rate and the compounding frequency.

a)  $r = 9\%, k = 2$

b)  $r = 11 \frac{1}{2} \%, k = 4$

We studied the continuously compounded formula for an investment earlier in lesson 27. It was given as  $A = Pe^n$ . For this lesson, it will be written  $S = Pe^n$ , where  $S$  is the final amount of the investment.

Ex 6: Jake has the option of investing \$1200 at an annual rate of 4.8% compounded quarterly or at an annual rate of 4.6% compounded continuously. Which would result in the best investment in a year's time?