

## Algorithm/Running Time Analysis

## Running Time

- Why do we need to analyze the running time of a program?
- Option 1: Run the program and time it
  - Why is this option bad?
  - What can we do about it?

## Pseudo-Code

- Used to specify algorithms
- Part English, part code

```
Algorithm (arrayMax(A, n))
maxVal = A[0]
for i=0 to n-1
    if maxVal < A[i]
        maxVal = A[i]
return maxVal
```

## Math Review

- Summation –  $\Sigma$
- Sum of  $n$  consecutive digits =  $n(n+1)/2$

## Counting Operations

```
Algorithm (arrayMax(A, n))
maxVal = A[0] //1
for i=0 to n-1 //n
    //2 op 2
    if maxVal < A[i]
        maxVal = A[i]
return maxVal //1
```

- Best case –  $n+2$
- Worst case –  $2n + 2$
- Average case – hard to analyze

## Asymptotic Notation

- $2n + 2$
- $n=5 \rightarrow 12$
- $n=100 \rightarrow 202$
- $n=1,000,000 \rightarrow 2,000,002$
- Running time **grows proportionally** to  $n$
- What happens as  $n$  gets large?

## Big-Oh

- $f(n)$  is  $O(g(n))$  if there is a real constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that  $f(n) \leq cg(n)$  for every integer  $n \geq n_0$
- $2n+2$  is  $O(n)$   $n_0 \geq 1$   $c=3$

## Examples

- $87n^4 + 7n$
- $3n \log n + 12 \log n$
- $4n^4 + 7n^3 \log n$

## Terminology

- Logarithmic –  $O(\log n)$
- Linear –  $O(n)$
- Linearithmic –  $O(n \log n)$
- Quadratic –  $O(n^2)$
- Polynomial –  $O(n^k)$   $k \geq 1$
- Exponential –  $O(a^n)$   $a > 1$

## Example

	0			$n-1$
0	6	4	...	2
	12	3	...	8
	...	...	...	...
$n-1$	3	8	...	1

- Find maximum number in  $n \times n$  matrix
- Algorithm:

## Example

- What is the big-oh running time of this algorithm?

```
Algorithm:
Input: A, n
curMax = A[0][0]
for i=0 i<n i++
  for j=0 j<n j++
    if curMax < A[i][j]
      curMax = A[i][j]
return curMax
```

## Another Example

0			$n-1$
2	4	...	8
0			$n-1$
6	8	...	3

- Determine how many elements of array 1 match elements of array 2
- Algorithm?

## Another Example

0			n-1
2	4	...	8

0			n-1
8	5	...	3

Algorithm:

Input: A, n, n

```
for i=0 to n-1
  for j=0 to n-1
    if A[i] == A[j]
      matches++
      break
```

- What is the running time of the algorithm?