

MA 15200 Lesson 33, Section 5.1 (part 2)

This is the first of two lessons of applied problems using a system of linear equations (linear system).

I Break-Even Analysis

The **Revenue** of a company is the money generated by sells of the product. The **Cost** of a company is the sum of any fixed costs and the cost of producing the product.

If a company produces and sells x number of a product, its **Revenue Function and Cost Function** are represented by

$$y = R(x) = (\text{price per unit})x$$

$$y = C(x) = \text{fixed cost} + (\text{cost per unit})x$$

These are both linear functions. In the revenue function, the price per unit is the slope. In the cost function, the cost per unit is the slope. When graphed, these lines will intersect. The **point of intersection** is called the **break-even point**. The x -coordinate will be the number of units of the product and the y -coordinate will be the amount of money coming in from sells and the amount of money going out for costs.

Ex 1: A manufacturer of CDs have determined that monthly fixed costs to make the CDs are \$337,500. It costs \$3.50 to make a CD. Each CD that is made can be sold for \$9.75. Find an equation to represent the costs of the manufacturer and an equation to represent to revenue where x represents the number of CDs made and sold during a month. What is the break-even point and what do the coordinates represent?

Ex 2: The weekly revenue of a company is represented by $y = R(x) = 12x$ and its weekly cost is represented by $y = C(x) = 5x + 12,000$. Find the value of $R(5,000) - C(5,000)$. What does this represent? Find the value of $R(800) - C(800)$. What does it represent?

The **Profit** of a company is found by subtracting costs from the revenue. If the profit is positive, there truly is a profit or gain. If it is negative, there is a loss for the company.

The **Profit Function** is found by $y = P(x) = R(x) - C(x)$.

Ex 4: The quarterly Cost function for a business that sells umbrellas (based on x umbrellas) is $C(x) = 9x + 26,500$. The quarterly Revenue function for this business is $R(x) = 21x$.

- a) Find the profit function, $P(x)$.
- b) How much profit (or lose) will the business incur during a quarter of the year, if they make and sell 4000 umbrellas?
- c) Find $P(1200)$ and describe what it means.
- d) How many umbrellas must the business make and sell quarterly in order to break even? Round to the nearest umbrella, if necessary.

Ex 3: A company that manufactures tricycles has a fixed monthly cost of \$25,000. It costs \$40 to produce each tricycle. The selling price of a tricycle is \$65.

- a) Write a cost function, letting x represent the number of tricycles made and sold during the month.
- b) Write a revenue function.
- c) Write a profit function.
- d) Determine the break-even point.

Ex 4: The percentage of Purdue students in group A (y) who like a certain TV show can be modeled by $x + 3y = 214$, where x is the number of weeks after the show premiered. The percentage of Purdue students in group B (y) who liked the TV show can be modeled by $2y - x = 136$, where x is the number of weeks after the show premiered. In what week after the premier was the percentage in both group A and group B the same? What is that percentage?

Ex 5: At a local fast food restaurant, the cost of two cheeseburgers and three fries was \$12.25. The cost of 5 cheeseburgers and two fries was \$21. Find the cost of a single cheeseburger and a single fries.

Ex 6: A 500 seat theater has tickets for sale at \$15 or \$25. How many tickets should be sold at each price to generate revenue of \$9200?