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Physics 8.01

Supplementary Notes 1: Dimensional Analysis

1.1 International System of System of Units

There are only four fundamental quantities (measurements) necessary to specify all physical phenomena: length, time, mass and charge. All other quantities are expressible in terms of these, constructed as a matter of convenience.

The basic system of units used throughout science and technology today is the internationally accepted *Système International* (SI) (Table 1). It consists of four base quantities and their corresponding base units: length (meter), mass (kilogram), time (second), and electric current (ampere). The unit for electric charge, the coulomb, is defined in terms of the ampere, and hence is referred to as a derived unit. In addition, three other quantities, temperature, amount of substance, and luminous intensity are part of the SI base quantities with corresponding units shown in Table 1.

Mechanics is based on just the first three of these quantities, the MKS or meter-kilogram-second system. An alternative metric system to this, still widely used, is the so-called CGS system (centimeter-gram-second). For distance and time measurements, British Imperial units (especially in the USA) based on the foot (ft), the yard (yd), the mile (mi), etc., as units of length, and also the minute, hour, day and year as units of time.

Table 1 Système International (SI) System of Units

Base Quantity	Base Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Electric Current	ampere (A)
Temperature	Kelvin (K)
Amount of Substance	mole (mol)
Luminous Intensity	candela (cd)

We shall refer to the dimension of the base quantity by the quantity itself, for example

$$\dim \text{length} \equiv [\text{length}] = L, \dim \text{mass} \equiv [\text{mass}] = M, \dim \text{time} \equiv [\text{time}] = T. \quad (1)$$

1.2 Dimensions of Commonly Encountered Quantities

Many physical quantities are derived from the base quantities by a set of algebraic relations defining the physical relation between these quantities. The dimension of the

derived quantity can always be written as a product of the powers of the dimensions of the base quantities.

Example 1 Derived Dimensions of Mechanical Quantities

(i) The dimensions of velocity are given by the relationship

$$[\text{velocity}] = [\text{length}]/[\text{time}] = \text{L} \cdot \text{T}^{-1}. \quad (2)$$

(ii) Force is also a derived quantity and using the definition of force $\vec{F} = m\vec{a}$ and acceleration $\vec{a} = d\vec{v} / dt$, force has dimensions

$$[\text{force}] = \frac{[\text{mass}][\text{velocity}]}{[\text{time}]} \quad (3)$$

We could express force in terms of mass, length, and time by the relationship

$$[\text{force}] = \frac{[\text{mass}][\text{length}]}{[\text{time}]^2} = \text{M} \cdot \text{L} \cdot \text{T}^{-2}. \quad (4)$$

(iii) The derived dimension of kinetic energy follows from the definition that $K = \frac{1}{2}mv^2$, thus

$$[\text{kinetic energy}] = [\text{mass}][\text{velocity}]^2, \quad (5)$$

which in terms of mass, length, and time is

$$[\text{kinetic energy}] = \frac{[\text{mass}][\text{length}]^2}{[\text{time}]^2} = \text{M} \cdot \text{L}^2 \cdot \text{T}^{-2} \quad (6)$$

(iv) The derived dimension of work is

$$[\text{work}] = [\text{force}][\text{length}], \quad (7)$$

which in terms of our fundamental dimensions is

$$[\text{work}] = \frac{[\text{mass}][\text{length}]^2}{[\text{time}]^2} = \text{M} \cdot \text{L}^2 \cdot \text{T}^{-2} \quad (8)$$

So work and kinetic energy have the same dimensions.

(v) Power is defined to be the rate of change in time of work so the dimensions are

$$[\text{power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{[\text{force}][\text{length}]}{[\text{time}]} = \frac{[\text{mass}][\text{length}]^2}{[\text{time}]^3} = M \cdot L^2 \cdot T^{-3} \quad (9)$$

In Table 2 we list the derived dimensions of some common mechanical quantities in terms of mass, length, and time.

Table 2 Dimensions of Some Common Mechanical Quantities

M ≡ mass, L ≡ length, T ≡ time

Quantity	Dimension	MKS unit
Angle	dimensionless ¹	Dimensionless = radian
Steradian	dimensionless	Dimensionless = radian ²
Area	L ²	m ²
Volume	L ³	m ³
Frequency	T ⁻¹	s ⁻¹ = hertz = Hz
Velocity	L · T ⁻¹	m · s ⁻¹
Acceleration	L · T ⁻²	m · s ⁻²
Angular Velocity	T ⁻¹	rad · s ⁻¹
Angular Acceleration	T ⁻²	rad · s ⁻²
Density	M · L ⁻³	kg · m ⁻³
Momentum	M · L · T ⁻¹	kg · m · s ⁻¹
Angular Momentum	M · L ² · T ⁻¹	kg · m ² · s ⁻¹
Force	M · L · T ⁻²	kg · m · s ⁻² = newton = N
Work, Energy	M · L ² · T ⁻²	kg · m ² · s ⁻² = joule = J
Torque	M · L ² · T ⁻²	kg · m ² · s ⁻²
Power	M · L ² · T ⁻³	kg · m ² · s ⁻³ = watt = W
Pressure	M · L ⁻¹ · T ⁻²	kg · m ⁻¹ · s ⁻² = pascal = Pa

¹ Even though angle and steradian are dimensionless quantities, it is often helpful to carry around a “unit” associated with them, like the radian, to understand their role in an expression or to determine if a result makes sense.