A Note on a Method for the Analysis of Significances en masse

PAUL SEEGER

University of Uppsala, Sweden

This note concerns the derivation of the p-mean significance levels, in the case of independent tests, for a mass-significance method developed by Eklund [1]. The solution is reached by formulating and solving an urn problem. Some comparisons are made with the p-mean significance levels of Duncan's multiple range test.

In three seminar papers from 1961–1963 [1] Eklund suggested the following solution to what he called the mass-significance problem: In large exploratory investigations it is desirable to keep the proportion of false significances low, at most equal to a small value k. Consider therefore the variable

$$y = \frac{\text{number of false significances}}{\text{number of significances}}$$

where the denominator is observed but the numerator has to be predicted. Both numerator and denominator are functions of the level of significance α', which is supposed to be used for each of N tests. Eklund's method consists in determining α' so that $y \leq k$, where k is predetermined. The observed number of significances at the level of α' may be denoted by $n(\alpha')$. Eklund considered three alternatives for the numerator. If the null hypothesis is true for each of the N tests we can predict the number of false significances to be $N\alpha'$. This is the most conservative of Eklund's alternatives. The method consists in finding a significance level α' for the individual test so that

$$\frac{N\alpha'}{n(\alpha')} \leq k$$

or

$$n(\alpha') \geq \frac{N\alpha'}{k} \tag{1}$$

Like the technique for making multiple comparisons based on Bonferroni's inequality [5], Eklund's method is only used to determine the level of significance for the individual test; it can be applied to N tests of any kind. One starts making the tests at the level $\alpha' = k$. If the criterion (1) is not satisfied, a lower value of

Received June 1967.

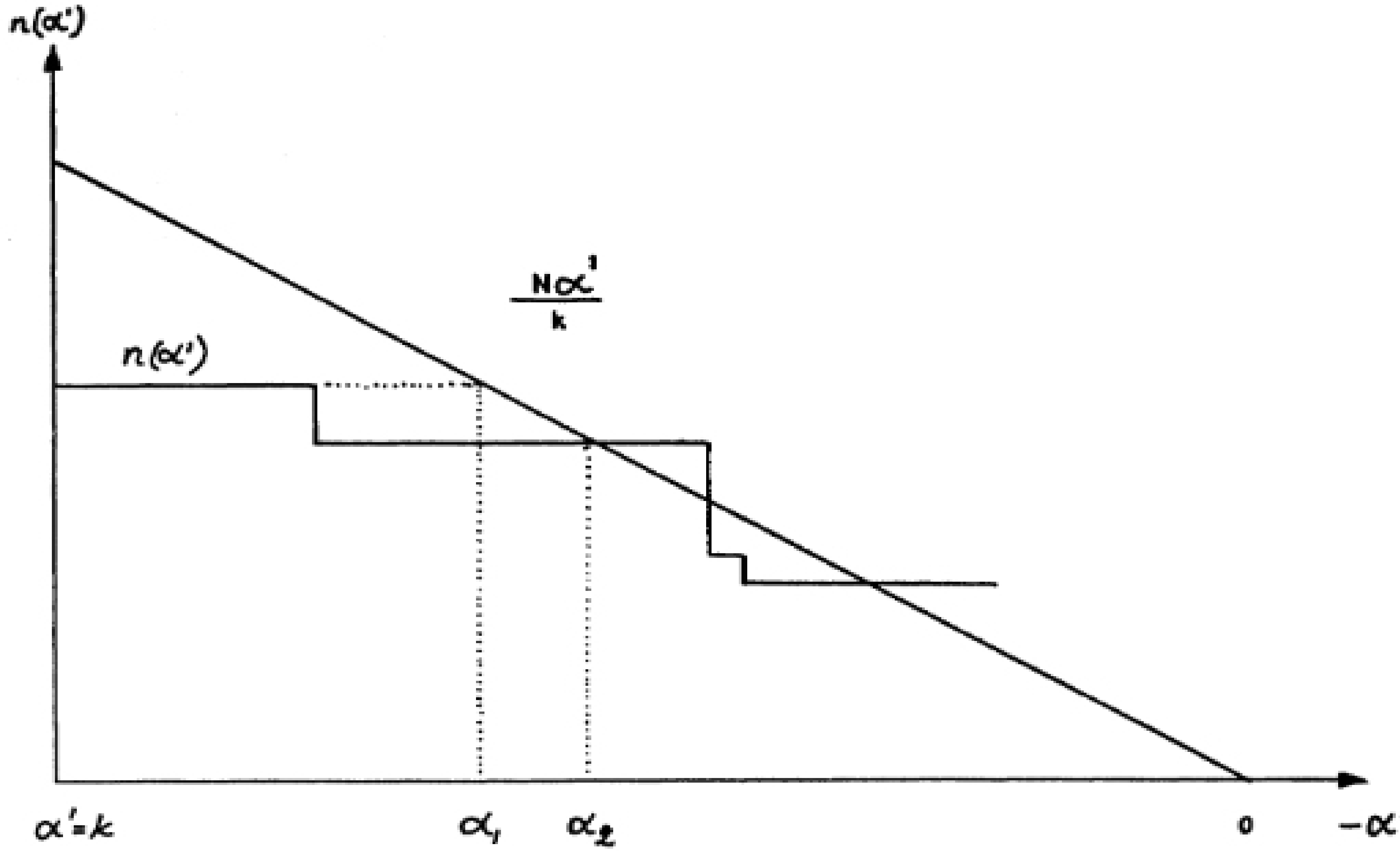

FIGURE 1
The number of significances $n(\alpha')$ plotted against the individual level of significance α'.

α' is tried until it is satisfied; that is until the curve in Fig. 1 touches the straight line. If this happens for the first time for $\alpha' = \alpha_0$, we say that we have obtained $n(\alpha_0)$ "mass-significances" ("approved" significances). In fact only a few α'-values need be investigated, as $n(\alpha')$ can not increase with decreasing α'. After a certain $\alpha' = \alpha_1$, where $n(\alpha_1) < N\alpha_1/k$ (and $n(\alpha') < N\alpha'/k$ for all $\alpha' > \alpha_1$), it is easy to compute the largest $\alpha' < \alpha_1$ which could give $n(\alpha_1) \geq n(\alpha') \geq N\alpha'/k$. Thus we investigate only $\alpha' = k$, α_1 and α_2 for the case of Fig. 1.

The individual significance levels of Eklund's method are determined not only by the number of tests, as, for example, in the method based on Bonferroni's inequality, but also by the observations themselves. This latter characteristic means that including some tests whose null hypotheses have very low prior probability of being true may increase the number of significances that exists among the original tests. However, requirement (1) seems to be a reasonable one, if measures are taken after every significant test (but not after non-significant ones). Only a small fraction of these measures are then unjustified.

As a referee has pointed out the properties of Eklund's method would certainly be greatly illuminated by a decision-theoretic discussion. In one of his alternatives Eklund, in a way, gave different prior probabilites to different null hypotheses when he assumed a certain (known) number $N_0 \leq N$ of them to be true. These aspects will not be considered here.

Eklund's method has also been discussed in a paper by Eklund and Seeger in 1965 [2] and in a monograph by Seeger 1966 [3]. In these publications the