Physics 1408-002
Principles of Physics

Lecture 18
– Chapter 11 & 12 –
March 24, 2009

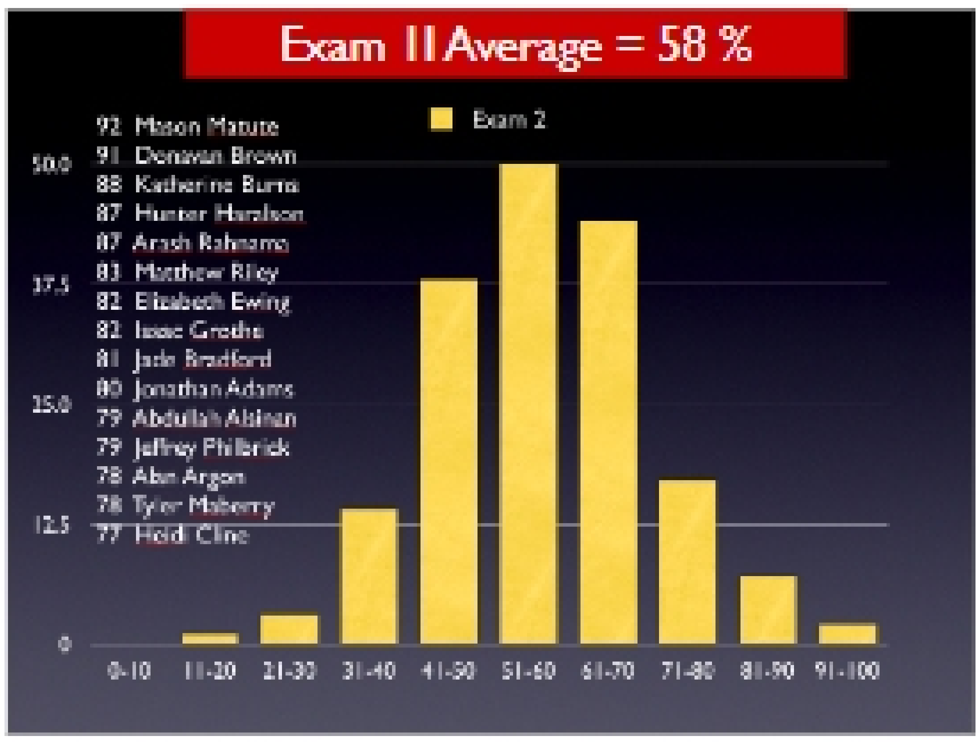
Sung-Won Lee
Sungwon.Lee@ttu.edu

Announcement I

Lecture note is on the web
Handout (6 slides/page)
http://highenergy.phys.ttu.edu/~slee/1408/

*** Class attendance is strongly encouraged and will be taken randomly. Also it will be used for extra credits.

HW Assignment #7 is placed on MateringPHYSICS , before spring break and is due by 11:59pm on Wendseday, 3/25



Chapter 11

Angular Momentum
General Rotation

- Angular Quantities
- Angular Momentum—Objects Rotating About a Fixed Axis
- Vector Cross Product; Torque as a Vector
- Angular Momentum of a Particle
- Angular Momentum and Torque
- Conservation of Angular Momentum
- The Spinning Top and Gyroscope
- Rotating Frames of Reference; Inertial Forces; The Coriolis Effect

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

The rotational analog of linear momentum is angular momentum, L: $L = I\omega$.

Then the rotational analog of Newton's second law is: $\Sigma\tau = \frac{dL}{dt}$.

In the absence of an external torque, angular momentum is conserved:

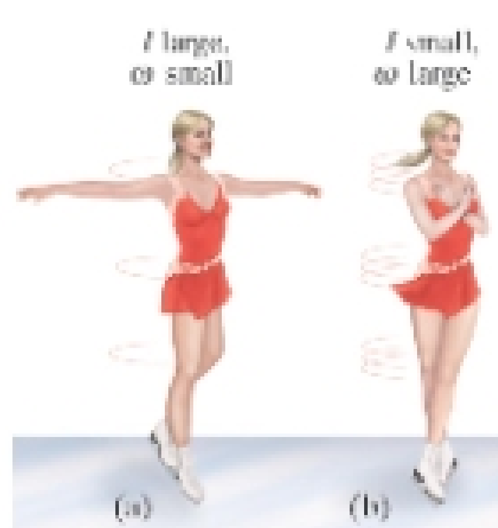
$$\frac{dL}{dt} = 0 \text{ and } L = I\omega = \text{constant.}$$

This means: $I\omega = I_0\omega_0 = \text{constant.}$

Therefore, if an object's moment of inertia changes, its angular speed changes as well.

11-1 Angular Momentum—Objects Rotating About a Fixed Axis

$$I\omega = I_0\omega_0 = \text{constant.}$$



A skater doing a spin on ice, illustrating conservation of angular momentum: (a) I is large and ω is small; (b) I is smaller so ω is larger.

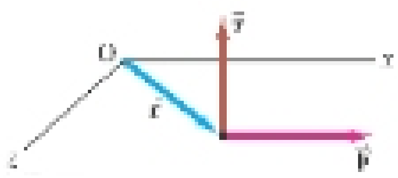
Angular momentum is conserved.

11-2 Vector Cross Product; Torque as a Vector

Torque can be defined as the vector product of the force and the vector from the point of action of the force to the axis of rotation:

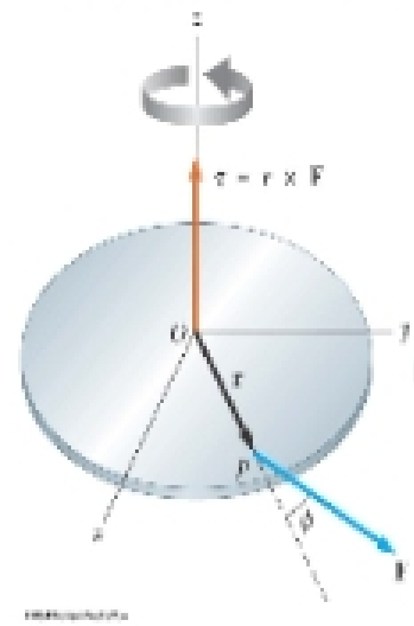
$$\vec{\tau} = \vec{r} \times \vec{F}.$$

Here, $\vec{r}$ is the position vector from the particle relative to O.



- The torque vector lies in a direction perpendicular to the plane formed by the position vector and the force vector

The Vector Product & Torque



$$C = |\vec{A} \times \vec{B}| = AB \sin\theta.$$

Using Determinants

- The cross product can be expressed as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{\mathbf{i}} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix}\hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{\mathbf{k}}$$

- Expanding the determinants gives

$$\mathbf{A} \times \mathbf{B} = \left(A_y B_z - A_z B_y\right)\hat{\mathbf{i}} - \left(A_x B_z - A_z B_x\right)\hat{\mathbf{j}} + \left(A_x B_y - A_y B_x\right)\hat{\mathbf{k}}$$

Torque Vector Example

- Given the force $\mathbf{F} = (2.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})$ N
  $\mathbf{r} = (4.00\,\hat{\mathbf{i}} + 5.00\,\hat{\mathbf{j}})$ m

$\tau = ?$

$$\tau = \mathbf{r} \times \mathbf{F} = [(4.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\mathrm{N}] \times [(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})\mathrm{m}]$$
$$= [(4.00)(2.00)\hat{\mathbf{i}} \times \hat{\mathbf{i}} + (4.00)(3.00)\hat{\mathbf{i}} \times \hat{\mathbf{j}}$$
$$+ (5.00)(2.00)\hat{\mathbf{j}} \times \hat{\mathbf{i}} + (5.00)(3.00)\hat{\mathbf{j}} \times \hat{\mathbf{j}}$$
$$= 2.0\hat{\mathbf{k}}\ \mathrm{N \cdot m}$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$
$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$
$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

11.3 Angular Momentum

- **Consider a particle of mass *m* located at the vector position r and moving with linear momentum p**

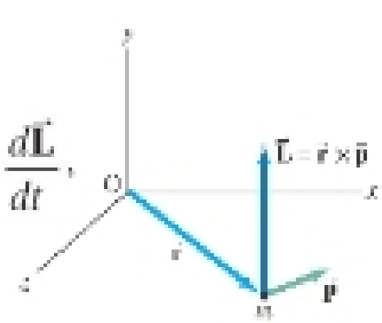
The angular momentum of a particle about a specified axis is given by: $\vec{L} = \vec{r} \times \vec{p}.$

If we take the derivative of $\vec{L}$, we find:

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}.$$

Since $\vec{r} \times \Sigma\vec{F} = \vec{r} \times \frac{d\vec{p}}{dt} = \frac{d\vec{L}}{dt},$

we have: $\Sigma\vec{\tau} = \frac{d\vec{L}}{dt}.$



Torque and Angular Momentum

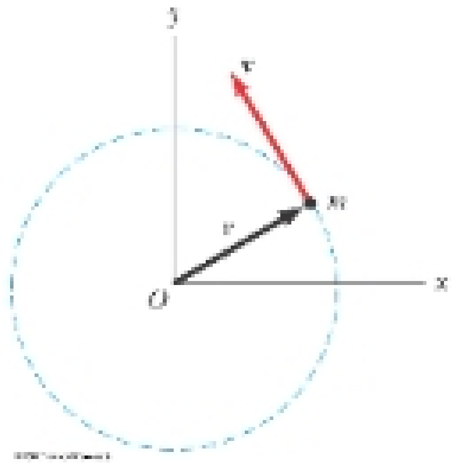
- The torque is related to the angular momentum
  - Similar to the way, force is related to linear momentum

$$L = r \times p \qquad \sum \tau = \frac{d\mathbf{L}}{dt}$$

- This is the rotational analog of Newton's 2nd Law
  τ and L must be measured about the same origin
- The SI units of angular momentum: [(kg·m²)/s]
- Both magnitude and direction of L depend on choice of origin
- The magnitude of $L = mvr \sin\phi$
  - ϕ is the angle between p and r
- The direction of L is perpendicular to the plane formed by r & p

Angular Momentum of a Particle

- The vector $L = r \times p$ is pointed out of the diagram
- The magnitude is
  $L = mvr \sin 90^{o} = mvr$
  $\sin 90^{o}$ is used
  since v is perpendicular to r
- A particle in uniform circular motion has a constant angular momentum about an axis through the center of its path



Angular Momentum of a System of Particles

- Total angular momentum of a system of particles is defined as the vector sum of the angular momenta of individual particles
  $L_{tot} = L_1 + L_2 + \ldots + L_n = \Sigma L_i$
- Differentiating with respect to time $\frac{d\mathbf{L}_{tot}}{dt} = \sum_i \frac{d\mathbf{L}_i}{dt} = \sum_i \tau_i$
- Any torques associated with the internal forces acting in a system of particles are zero.

$$\sum \tau_{ext} = \frac{d\mathbf{L}_{tot}}{dt}$$

11.3 L of a Rotating Rigid Object

- Each particle of the object rotates in the *xy* plane about the *z*-axis w/ angular speed of ω
- The angular momentum of an individual particle:
- L and ω are directed along the *z*-axis

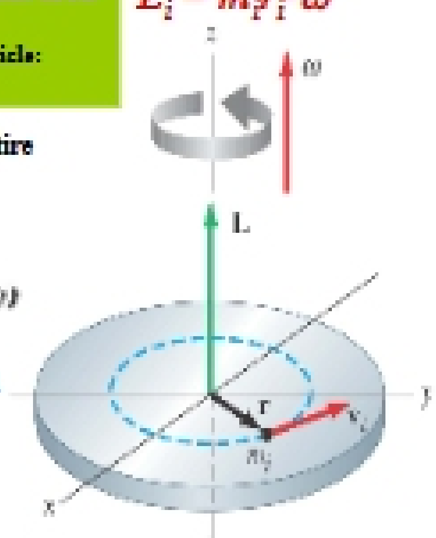
$$L_i = m_i r_i^2 \omega$$

- To find the angular momentum of the entire object, add the angular momenta of all the individual particles

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = I\omega$$

- This also gives the rotational form of Newton's 2nd-Law

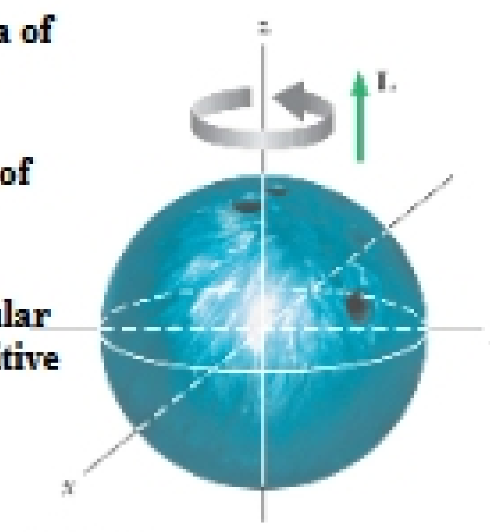
$$\sum \tau_{ext} = \frac{dL_z}{dt} = I\frac{d\omega}{dt} = I\alpha$$



$$L_\omega = I\omega.$$

Angular Momentum of a Bowling Ball

- The momentum of inertia of the ball: $I = 2/5MR^2$
- The angular momentum of the ball: $L_z = I\omega$
- The direction of the angular momentum is in the positive *z*-direction



11.4 Conservation of Angular Momentum

- The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is "0".
  - Net torque = 0 -> means that the system is isolated!!
- $\mathbf{L}_{tot}$ = constant or $\mathbf{L}_i = \mathbf{L}_f$
- For a system of particles, $\mathbf{L}_{tot} = \Sigma \mathbf{L}_n$ = constant
- If the mass of an isolated system undergoes redistribution, the moment of inertia (I) changes.
  - The conservation of angular momentum requires a compensating change in the angular velocity.
  - $I_i \omega_i = I_f \omega_f$
    - This holds for rotation about a fixed axis and for rotation about an axis through the center of mass of a moving system
    - The net torque must be zero in any case

11-5 Angular Momentum and Torque for a Rigid Object

An Atwood machine consists of two masses, m_A and m_B, which are connected by an inelastic cord of negligible mass that passes over a pulley.

If the pulley has radius R_0 and moment of inertia I about its axle, determine the acceleration of the masses m_A and m_B, and compare to the situation where the moment of inertia of the pulley is ignored.