Math 128
Midterm Examination 1 – September 23, 2008

Name _____

6 problems, 100 points.

Instructions: Show all work – partial credit will be given, and "Answers without work are worth credit without points." You don't have to simplify your answers. You may use a simple calculator that is not graphing or programmable. You may have a 3x5 card, but no other notes.

1. *(18 points) Compute the following partial derivatives*

   (a) h_z, *where* $h(x,y,z) = \dfrac{x^2+y^2+z^2}{xyz}$

   Using the quotient rule,

   $$h_z = \frac{2z \cdot xyz - (x^2+y^2+z^2)xy}{(xyz)^2}.$$

   (b) $\dfrac{\partial}{\partial x} \ln(e^{x^2y} + e^{y^2x})$

   Using the chain rule twice,

   $$\frac{\partial f}{\partial x} = \frac{1}{e^{x^2y} + e^{y^2x}} \cdot (e^{x^2y} \cdot 2xy + e^{y^2x} \cdot y^2).$$

   (c) f_{xy}, *where* $f(x,y) = 3\sin xy$

   We first compute f_x:

   $$f_x = 3y\cos xy.$$

   And then $f_{xy} = f_{yx}$:

   $$f_{xy} = 3\cos xy - 3xy\sin xy.$$

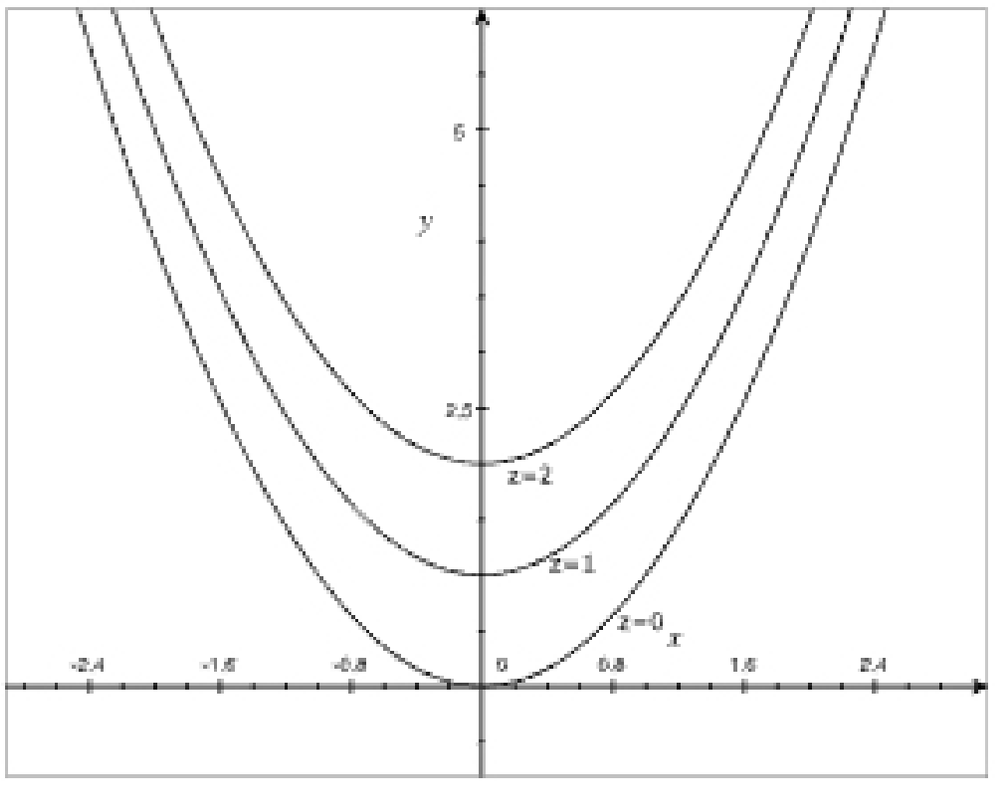
2. *(12 points) Sketch the level curves of* $z = y - x^2$ *at the levels* $z = 0, 1, 2$. *Make sure to label your graph.*

   We graph

   $$0 = y - x^2, \quad 1 = y - x^2, \quad 2 = y - x^2,$$

which, after moving the y's to the right hand side and multiplying by -1 are

$$y = x^2, \quad y = x^2 + 1, \quad y = x^2 + 2.$$



3. *(20 points) Let* $f(x, y) = x^2 + 4y^3 - 6xy + 10$.

   (a) *Find the partial derivatives* $\dfrac{\partial f}{\partial x}$ *and* $\dfrac{\partial f}{\partial y}$.

   $\dfrac{\partial f}{\partial x} = 2x - 6y$, $\dfrac{\partial f}{\partial y} = 12y^2 - 6x$.

   (b) *Find the critical points for* f.

   We set $2x - 6y = 0$ and $12y^2 - 6x = 0$. Dividing the first equation by 2 and moving y to the rights gives us $x = 3y$. Substituting this into the other equation gives us $12y - 6x = 12y^2 - 18y = 0$. We factor this last equation as $6y(2y - 3) = 0$. Thus, $y = 0$ or $y = \frac{3}{2}$, and since $x = 3y$, we have the critical points $(0, 0)$ and $(\frac{9}{2}, \frac{3}{2})$.

   (c) *Calculate the 2nd derivatives* $\dfrac{\partial^2 f}{\partial x^2}$, $\dfrac{\partial^2 f}{\partial y^2}$, *and* $\dfrac{\partial^2 f}{\partial x \partial y}$.

   $$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} 2x - 6y = 2$$
   $$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} 12y^2 - 6x = -6$$
   $$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} 12y^2 - 6x = 24y.$$

(d) *Using the 2nd derivative test, determine which points are relative maxima, relative minima, and saddle points.*

From the calculation of the 2nd derivative, we have the discriminant

$$D(x,y) = f_{xx}f_{yy} - (f_{xy})^2 = 48y - 36.$$

Plugging in the critical points, we get that
$D(0,0) = 0 - 36 = -36 < 0$, hence $(0,0)$ is a saddle point.

$D(\frac{9}{2}, \frac{3}{2}) = 48 \cdot \frac{3}{2} - 36 = 36 > 0$, and since $f_{xx} = 2 > 0$, that $(\frac{9}{2}, \frac{3}{2})$ is a relative minimum.

4. *A small company has a local monopoly on two competing products: model houses, and model hotels. It costs them \$200 to build each model house, \$300 for each hotel. The total revenue from selling x houses and y hotels is $1000x + 1200y - 2xy - x^2 - 2y^2$.*

   (a) *(5 points) What is the profit $P(x,y)$ from selling x houses and y hotels?*

   Profit = Revenue - Cost, thus
   $P(x,y) = 1000x + 1200y - 2xy - x^2 - 2y^2 - (200x + 300y) = 800x + 900y - 2xy - x^2 - 2y^2$.

   (b) *(5 points) Calculate the partial derivatives P_x and P_y.*
   $P_x = 800 - 2x - 2y$
   $P_y = 900 - 2x - 4y$.

   (c) *(5 points) Find the number of houses and hotels the company should build to maximize their profit.*
   We set $P_x = P_y = 0$ and solve for x and y. First: $P_x = 0 = 800 - 2x - 2y$. When we divide by 2 and move the $2x$ and $2y$ to the other side, we get $x + y = 400$, or $x = 400 - y$. Then: $P_y = 0 = 900 - 2x - 4y$. But we plug in $x = 400 - y$ to give $0 = 900 - 800 + 2y - 4y = 100 - 2y$. Thus, $100 = 2y$, or $y = 50$ and $x = 400 - y - 350$.

   A critical point occurs at 50 hotels and 350 houses.

   (d) *(3 points) Explain briefly why your answer in (c) is a maximum.*
   We didn't get the tools for a complete answer to this question. I gave full credit for a variety of answers. For example: