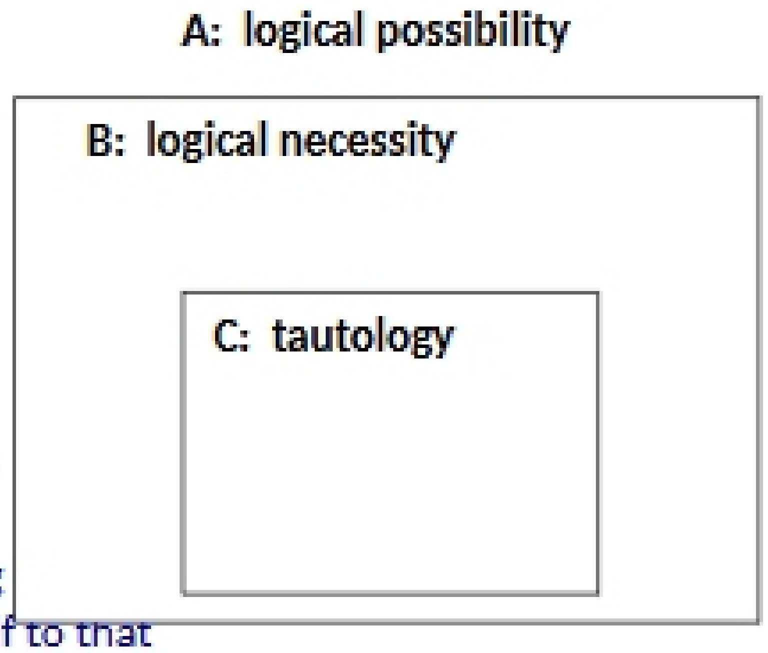
Quiz #4 **ANSWER KEY** with comments Name ________________________________
PHIL 110, Spring 2014 (*12 points*)
Hamilton
2/19/2014

Part 1: In the blank next to each of the following sentences, put the *one capital letter* (from the accompanying diagram) that labels the <u>innermost</u> logical region to which the sentence belongs. (*4 points per blank*)



1. SameRow(a,b) ___A____

   Two objects *a* and *b* can be in the same row, but they don't *have* to be. This is just a possibility.

2. SameRow(a,a) ___B____
   An object *a* must be in the same row as itself, so this one is a logical necessity. However, it's not a tautology because the meaning of the predicate SameRow is crucial to making the sentence logically necessary. And if you blinded yourself to that meaning by replacing the atomic sentence with a capital letter like P, you'd just have P, and without knowing what P means, you wouldn't be able to tell if it had to always be true in every world.

3. SameRow(a,b) v SameSize(a,b) ___A____
   This is only a logical possibility because it could be false in some worlds, namely, any worlds in which objects *a* and *b* are neither the same size nor in the same row.

4. SameRow(a,a) ∧ SameRow(a,a) ___B____
   This one is a logical truth for the same reason as #2 above. Conjoining another SameRow(a,a) is simply redundant; it doesn't really add or subtract anything from the necessity of the sentence. And the sentence is not a tautology because if you blind yourself to the meaning of the atomic sentences you get simply P ∧ P, which is no more necessarily true than is simply P. Without knowing what P means, you can't tell if it must be true in all worlds or not.

5. SameRow(a,a) ∧ SameSize(a,a) ___B____
   This one is logically necessary for reasons similar to #4 above. An object *a* is necessarily the same size as itself and in the same row as itself, so both halves of the conjunction must always be true, making the entire conjunction always true. It's not a tautology, though, for when you blind yourself to the meaning of the atomic sentences you get something like P ∧ Q, and you can't know if that sentence would necessarily be true without knowing what P and Q individually mean.

6. SameRow(a,b) v SameRow(b,a) ___A____
   Don't let this one fool you. For the disjunction to be true, at least one of the disjuncts (halves) must be true … so the question is whether you can think of any situation in which *both* disjuncts would

ever be *false*. Of course you can: any world in which objects *a* and *b* are *not* in the same row. In such a world, the sentence is false, which shows that this sentence is no more than a logical possibility.

7. ¬(Cube(a) ∧ ¬Cube(a)) ____C____
This sentence can never be false. Think about it: Inside the outer parentheses there is an illogical statement that makes no sense, namely, that object *a* is both a cube *and not* a cube at the same time. That's crazy. But then this crazy statement is negated, which is to say that it is false. And, yes, a crazy contradictory statement like what's inside the parentheses will be false in every world, which means that the *negation* of that always-false statement will *always be true*. So this sentence is a logical necessity. But it is more than that, it is also a tautology. Put your blinders on and you get ¬(P ∧ ¬P) which, if you think about it, is clearly just as logically necessary as was the original version. It doesn't matter what P means; the affirmation and the negation of P cannot both be true at the same time, so the negation of that contradictory statement will always be true.

BONUS QUESTION (*4 points*):

8. ¬(SameSize(a,b) ∧ ¬SameSize(b,a)) ____B____
This one is very similar to #7 above, and it is logically necessary for the same reason (i.e., the *negation* of a contradictory statement will always be true). However, this one is not a tautology for a simple reason: notice that the order of the arguments of the predicate differs between the two atomic sentences (i.e., [a,b] versus [b,a]), and this is enough to make these two distinct atomic sentences (even though, of course, they have the same meaning). So, when you blind yourself to the meanings of the atomic sentences, you get ¬(P ∧ ¬Q) which doesn't have the same logical necessity as the original.

Part 2: For each of the following sentences of FOL, <u>circle the letter</u> next to the English translation that *best* captures the meaning of the FOL sentence. (*4 points each*)

9. ¬(RightOf(d,e) ∧ RightOf(e,d))

    a. *d* is not right of *e*, *and e* is not right of *d*.

    b. It's not the case *both* that *d* is right of *e and* that *e* is right of *d*.

    c. It's not the case that *d* is right of *e* and it's *also* not the case that *e* is right of *d*.

    Choices (a) and (c) here say essentially the same thing, and both get the translation wrong because both suggest that each of the atomic sentences RightOf(d,e) and RightOf(e,d) are negated individually. However, in reality it is only the *conjunction* of the two atomic sentences that is negated (i.e., they can't *both* be true), leaving open the possibility that just one or the other of them may be true.

10. (Large(d) ∨ Med(d)) ∧ Tet(d)

a. *d* is a large or medium tetrahedron.

b. Either *d* is large or it's a medium tetrahedron

c. *d* is both large and medium, or it's a tetrahedron.

Notice that the main connective in 10 is the conjunction, which means that *d* must be a tetrahedron for the sentence to be true, and also that this tetrahedron must be either large or medium (so that the first conjunct of the sentence will also be true—which it must be if the entire conjunction is true). Only answer choice (a) gives this meaning.

11. ¬(RightOf(e, g) ∧ LeftOf(e, f)) ∧ ¬(RightOf(d, g) ∧ LeftOf(d, f))

a. Either *e* or *d* is not right of *g* and not left of *f*.

b. Neither *e* nor *d* is right of *g* and left of *f*.

c. *e* is not right of *g*, and *e* is not left of *f*, and *d* is not right of *g*, and *d* is not left of *f*.

Sentence 11 is a conjunction of two negated phrases, meaning that both of the negated phrases must be true. For that to be the case, then *e* cannot be both right of *g* and left of *f*, and likewise *d* cannot be both right of *g* and left of *f*. Choice (b) is the only one that captures this meaning.