Quiz #6 -- ANSWER KEY with comments
PHIL 110, Spring 2014
HAMILTON, 3/3/2014

Name____
(*10 pts*)

PART 1: Circle TRUE or FALSE for each of the following statements. (*3 pts each*)

1. T — A valid argument may have premises that contradict each other.
   Not only may a valid argument have contradictory premises, but *any* argument with contradictory premises will *automatically* be valid, because anything and everything follows from (i.e., is a logical consequence of) a contradiction. Refer to the discussion of why the proof rule ⊥ **Elim works.**

2. F — The proof rule ⊥ **Intro** is also known as "proof by contradiction".
   Actually, it's the proof rule ¬ **Intro** that is known as proof by contradiction.

3. F — The proof rule ¬ **Elim** involves the use of a subproof, the premise of which is the opposite (or negation) of what you actually want to prove.
   No, this actually describes to the proof rule ¬ **Intro**.

4. F — The schema for the use of the proof rule ⊥ **Elim** involves first reaching a contradiction as the conclusion of a subproof, then closing the subproof and asserting anything at all one level back (toward the main proof level).
   It's the last part of this description that makes it false. In actuality, with the proof rule ⊥ **Elim** you are allowed to assert 'anything at all' only *at the same proof level as* (i.e., directly underneath) the contradiction symbol, *not* 'one level back'. (Don't confuse this rule with ¬ **Intro**, which also involves the use of a contradiction symbol. With ¬ **Intro,** you do close the subproof immediately after the contradiction symbol and then assert the negation of the premise one level back.)

5. T — Once a subproof has been discharged, only the subproof as a whole may be cited as input to a proof rule.
   That's right, because the entire subproof hangs on its premise, meaning that anything stated within the subproof is 'true' only if the premise of the subproof is true. In most cases you'll either *not know* whether the premise of the subproof is actually true (as with some or all of the premises of subproofs used with a proof by cases—i.e., disjunction elimination—strategy) or you'll actually know that the premise you're 'supposing' is, in fact, *not* true at all (as in all cases of the subproof used with a proof by contradiction—i.e., negation introduction—strategy).

Part 2: Compose a proof demonstrating the validity of the following argument. Be sure to format the proof correctly, including providing any necessary fitch bars, numbering your steps, and justifying each step (as needed) with a specific rule and any required step citations. *(partial credit up to 15 pts)*

6. PREMISE: **Cube(b) ∧ Dodec(c)**
   CONCLUSION: **¬¬Dodec(c)**

1	Cube(b) ∧ Dodec(c)	
2	Dodec(c)	∧ Elim: 1
3	¬Dodec(c)	
4	⊥	⊥ Intro: 2,3
5	¬¬Dodec(c)	¬Intro: 3-4

Many of you made the above proof harder than it actually is. The first key was to realize that the only 'piece' of the main premise that you need is the final conjunct **Dodec(c)**, which you can access only by eliminating the conjunction in which it's embedded (you could either assert this individual conjunct at the main proof level before starting the subproof, as I've shown above, or you could wait and assert it inside the subproof right before the contradiction symbol, as some of you did). The other key was to recognize that the only way to generate the double negation in the goal is to use a proof by contradiction strategy via the rule **¬ Intro.** If you remembered how that rule works, you knew that you would need to premise the *opposite* of the goal and show that this premise leads to a contradiction, so that you could then close the subproof and assert the negation of the premise back at the main level. The subproof premise you needed for this is **¬Dodec(c),** which when negated at step 5 becomes the desired **¬¬Dodec(c).** Some of you were on the right track but used **Dodec(c)** as the premise of your subproof (i.e., without the negation)—but that won't work, because you needed the *opposite* of the goal you want to prove, whereas Dodec(c) is instead *equivalent* to the goal (not it's opposite). Perhaps those of you who made that mistake were reluctant to premise something you knew to be false in light of the main premise—but keep in mind that this is precisely how proof by contradiction (negation intro) works: the premise of your subproof will be the opposite of the goal you're actually working toward, which means you intentionally premise something that you know will lead to a contradiction ... something that you know is actually false. Finally, a few students tried opening two subproofs, as though attempting a 'proof by cases' strategy. But notice that step 1 is a conjunction, not a disjunction, so you can't base a proof by cases off of it.

PART 3: Fill in the missing rules and step citations for the following proof. Only fill in those blanks that require a rule or step citation (not all of the blanks should be filled in). (*2 pts each blank*)

7.

1	P v Q v R	*rule?* ___	*step(s)?* ___
2	¬¬¬P ∧ ¬¬¬R	*rule?* ___	*step(s)?* ___
3	P	*rule?* ___	*step(s)?* ___
4	¬¬¬P	*rule?* ∧ Elim	*step(s)?* 2
5	¬P	*rule?* ¬ Elim	*step(s)?* 4
6	⊥	*rule?* ⊥ Intro	*step(s)?* 3, 5
7	Q	*rule?* ⊥ Elim	*step(s)?* 6
8	Q	*rule?* ___	*step(s)?* ___
9	Q	*rule?* Reit	*step(s)?* 8
10	R	*rule?* ___	*step(s)?* ___
11	¬¬¬R	*rule?* ∧ Elim	*step(s)?* 2
12	¬R	*rule?* ¬ Elim	*step(s)?* 11
13	⊥	*rule?* ⊥ Intro	*step(s)?* 10, 12
14	Q	*rule?* ⊥ Elim	*step(s)?* 13
15	Q	*rule?* v Elim	*step(s)?* 1, 3-7,8-9, 10-14

The overall structure of the above proof is a proof by cases (hence, the premises of the three subproofs are taken from the three disjuncts of the main premise, and the rule **v Elim** justifies the last step of the proof). Some students (despite my many warnings) tried to justify the premises in the proof—especially the premises of the subproofs at steps 3, 8, and 10. But remember that premises of any sort are *never* justified (they are simply 'supposed' or 'given' to be true). If you had trouble with parts of this proof, study the rules and step citations above carefully to understand how each step is justified, and refer back to the explanations of each rule in the lessons as necessary. You can also shoot me an email with specific questions.