CMSC250, Spring 2004 — Homework 2 Answers

Due Wednesday, February 11 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. For each of the following statements, give its converse, inverse, and contrapositive in English sentences; be sure to label the three parts of each answer. You may change verb tenses to make your answers sound better.

   (a) "If one thinks, one must reach conclusions."[1]

   Answer:
   - Converse: If one must reach conclusions, then one thinks.
   - Inverse: If one does not think, then one must not reach conclusions.
   - Contrapositive: If one must not reach conclusions, then one does not think.

   (b) "If you look good and dress well, you don't need a purpose in life."[2]

   Answer:
   - Converse: If you don't need a purpose in life, then you look good and dress well.
   - Inverse: If you do not look good or do not dress well, then you need a purpose in life.
   - Contrapositive: If you need a purpose in life, then you do not look good or do not dress well.

   (c) "California is a fine place to live if you happen to be an orange."[3]

   Answer:
   - Converse: If California is a fine place to live, then you happen to be an orange.
   - Inverse: If you don't happen to be an orange, then California is not a fine place to live.
   - Contrapositive: If California is not a fine place to live, then you don't happen to be an orange.

   (d) "You can be free only if I am free."[4]

   Answer:
   - Converse: If I am free, then you can be free.
   - Inverse: If you cannot be free, then I am not free.
   - Contrapositive: If I am not free, then you cannot be free.

2. Construct a complete truth table to help you determine if the following argument is valid or not. State whether it is valid or not, indicate the entries in the truth table that led you to your answer, and explain why those entries support your answer.

$$\begin{array}{ll} & p \to (q \vee r) \\ & q \to \sim p \\ \hline \therefore & p \to r \end{array}$$

[1] Helen Keller (1880–1968)
[2] Robert Pante
[3] Fred Allen (1894–1956)
[4] Clarence Darrow (1857–1938)

Answer:

p	q	r	$\sim p$	$q \vee r$	Premise $p \to (q \vee r)$	Premise $q \to \sim p$	Conclusion $p \to r$	
1	1	1	0	1	1	0	1	
1	1	0	0	1	1	0	0	
1	0	1	0	1	1	1	1	← Critical row
1	0	0	0	0	0	1	0	
0	1	1	1	1	1	1	1	← Critical row
0	1	0	1	1	1	1	1	← Critical row
0	0	1	1	1	1	1	1	← Critical row
0	0	0	1	0	1	1	1	← Critical row

The critical rows are the rows where all the premises are true. Since in all five critical rows the conclusion is also true, this argument is valid.

3. Use the rules of inference you were given to complete the two proofs below. Use the same format as was shown in class for these proofs — each line of your proof must be justfied with the rule and line numbers you used to obtain that line.

(a)

P1	$\sim x \vee w$
P2	$(x \to y) \to (s \to z)$
P3	$\sim z$
$\therefore$	$s \to w$

Answer:

Line	Statement	Rule	Lines Used
1	$\sim (x \to y) \vee (s \to z)$	Definition of $\to$	P2
2	$(x \wedge \sim y) \vee (\sim s \vee z)$	Definition of $\to$	1
3	$(x \wedge \sim y) \vee \sim s \vee z$	Associativity	2
4	$(x \wedge \sim y) \vee \sim s$	Disjunctive syllogism	3, P3
5	s	Assume	—
6	$x \wedge \sim y$	Disjunctive syllogism	4,5
7	x	Conjunctive simplification	6
8	w	Disjunctive syllogism	P1, 7
9	$s \to w$	Closing conditional world	5–8

Here's another way.

Line	Statement	Rule	Lines Used
1	$\sim (x \to y) \vee (s \to z)$	Definition of $\to$	P2
2	$s \wedge \sim w$	Assume	—
3	$\sim w$	Conjunctive simplification	2
4	$\sim x$	Disjunctive syllogism	P1, 3
5	$\sim x \vee y$	Disjunctive addition	4
6	$x \to y$	Definition of $\to$	5
7	$\sim\sim (x \to y)$	Double negation	6
8	$s \to z$	Disjunctive syllogism	1, 7
9	s	Conjunctive simplification	2
10	z	Modus ponens	8, 9
11	$z \wedge \sim z$	Conjunctive addition	10, P3
12	$\sim (s \wedge \sim w)$	Closing cond world w/ contra	2–11
13	$\sim s \vee \sim\sim w$	DeMorgan's law	12
14	$\sim s \vee w$	Double negative	13
15	$s \to w$	Definition of $\to$	14

And yet another way.

Line	Statement	Rule	Lines Used
1	s	Assume	—
2	$x \to y$	Assume	—
3	$s \to z$	Modus ponens	2, P2
4	z	Modus ponens	3, 1
5	$z \wedge \sim z$	Conjunctive addition	P3, 4
6	$\sim (x \to y)$	Closing cond world w/ contra	2–5
7	$x \wedge \sim y$	Definition of $\to$	6
8	x	Conjunctive simplification	7
9	$\sim\sim x$	Double negation	8
10	w	Disjunctive syllogism	P1, 9
11	$s \to w$	Closing conditional world	1–10

(b)

P1	$(a \wedge \sim b) \vee (c \wedge a)$
P2	$(a \vee d) \to \sim f$
P3	$c \to (f \vee \sim a)$
$\therefore$	$\sim b$

Answer:

Line	Statement	Rule	Lines Used
1	$(a \wedge \sim b) \vee (a \wedge c)$	Commutativity	P1
2	$a \wedge (\sim b \vee c)$	Distributive law	1
3	a	Conjunctive simplification	2
4	$\sim b \vee c$	Conjunctive simplification	2
5	$a \vee d$	Disjunctive addition	3
6	$\sim f$	Modus ponens	5, P2
7	$\sim f \wedge a$	Conjunctive addition	6, 3
8	$\sim (f \vee \sim a)$	Double negative, DeMorgan's law	7
9	$\sim c$	Modus tollens	P3, 8
10	$\sim b$	Disjunctive syllogism	9, 4