Problem Set 1 - *Answers*
International Equilibrium

1. a. Explain how it is possible for a production function to have the property of constant or increasing returns to scale and still satisfy the Law of Diminishing Returns.

Returns to scale refers to what happens to output when all inputs are expanded in the same proportion, while the Law of Diminishing Returns refers to expansion of one input holding another input fixed. Thus, the Law of Diminishing Returns says that expansion of the labor input, say, holding the capital input fixed, causes output to rise by smaller and smaller amounts, so that at least eventually the rise in output is smaller than proportional to the expansion of labor. But if that increase in labor were instead accompanied by an increase in capital also, the latter would increase output by more. This would make it possible for output to rise by an equal or greater proportion than the increase in both inputs, thus displaying constant or increasing returns to scale.

b. If a production function has only one input, say labor, and displays constant returns to scale, does it then violate the Law of Diminishing Returns?

With constant returns to scale and only labor as an input, output is simply proportional to the amount of the input. It is true, therefore, that the marginal product of labor is then constant and does not diminish as more and more labor is used. Whether that is a violation of the Law of Diminishing Returns, however, is debatable, since we are not fixing the input of any other factor, there not being any other factor available to fix.

c. Suppose that a production function has two inputs, K and L, and that, contrary to the Law of Diminishing Returns, an increase in L alone always causes output to rise by the same proportion that L has increased. Show that the production function must therefore display increasing returns to scale.

If K and L both rise by some proportion λ, we can break the effect into two parts, the effect of L alone rising by λ, and then the effect of K rising by λ. By assumption, the first of these causes output to rise by λ. If the marginal product of K is positive, then the rise in K causes an additional increase in output, so that the total rise in output is then more than λ. But that is the definition of increasing returns to scale. (You can also show this more formally by differentiating the production function.)

d. Differentiate the definition of "Homogeneous of Degree *k*" with respect to λ, and evaluate the result at λ=1. Then use this result to show that, if factors of production are paid the value of their marginal products as assumed under perfect competition, then factor payments will exactly equal the value of output if returns to scale are constant, but will exceed it if returns to scale are increasing. What does the latter result tell you about the compatibility of perfect competition with increasing returns to scale?

The definition of homogeneity of degree k is

$$F(\lambda K, \lambda L) = \lambda^k F(K, L)$$

Differentiating with respect to λ gives

$$F_K K + F_L L = k\lambda^{k-1} F(K, L) = kF(K, L) \quad at\ \lambda = 1$$

With competitive factor pricing, $r = pF_K$ *and* $w = pF_L$. *If we multiply the above equation through by p, substitute into it these factor prices, and replace* $F(K, L)$ *with X, we get*

$$rK + wL = kpX$$

If returns to scale are constant, k=1, and this says that the market value of the inputs equals the value of the output. If returns to scale are increasing, then k>1 and the market value of the inputs is greater than the value of the output.

This suggests, correctly, that perfect competition is inconsistent with increasing returns to scale, since a competitive firm would not have enough revenue to pay competitive prices for its inputs.

2. a. In the 2×2 production model with constant returns to scale, show that if the two industries have the same factor intensity for all factor prices, then the production possibility frontier (PPF) is a straight line.

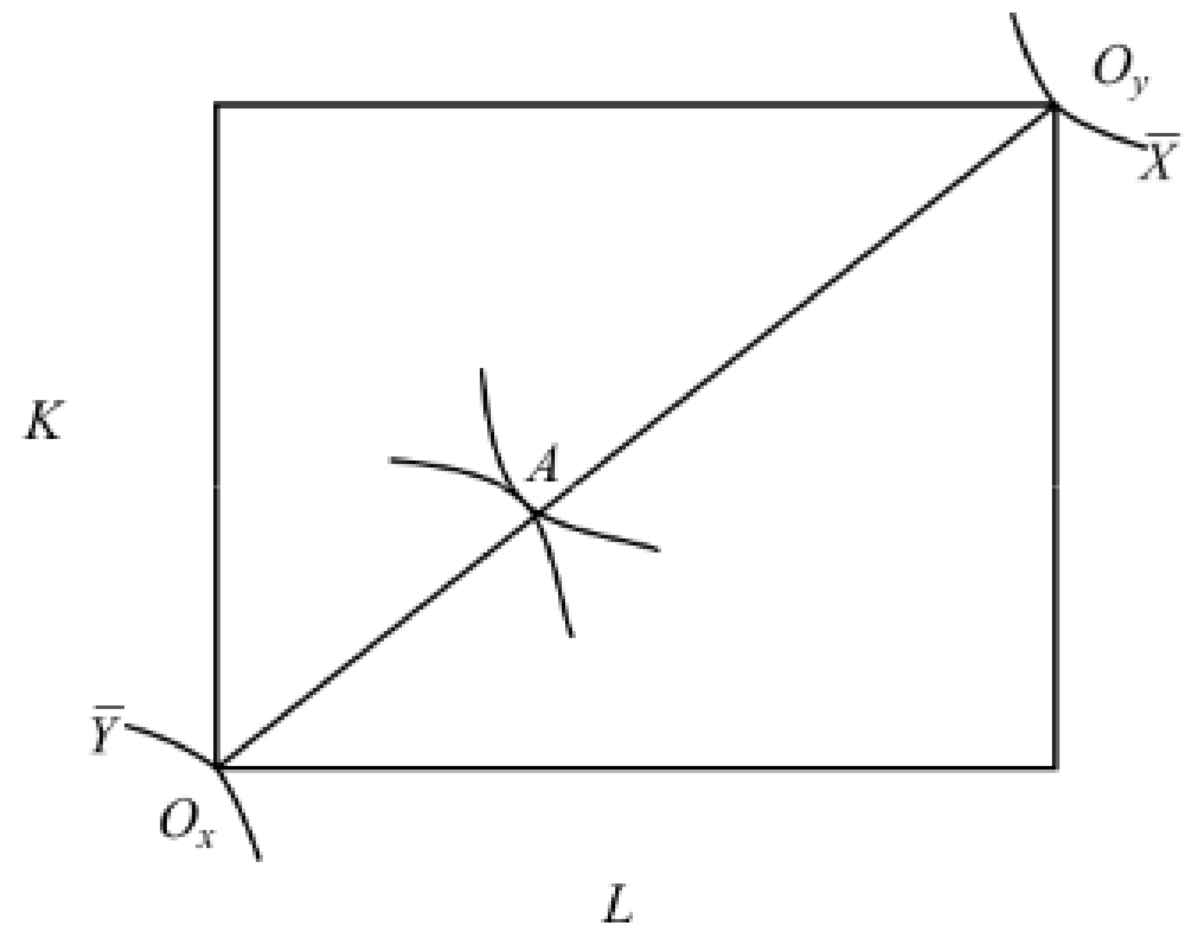
If the two industries have the same factor intensities – that is, when they face the same factor prices they employ the same ratios of the factors – then the efficiency locus is the diagonal of the Edgeworth Box. This can be seen by contradiction from the more usual case where the efficiency locus is not the diagonal, as in Figure 2.8 of the textbook. At any point along that efficiency locus, the isoquants are tangent and thus the industries face the same factor prices. But being off the diagonal, one industry necessarily has a capital-labor ratio smaller than the slope of the diagonal while the other has a capital-labor ratio larger than the diagonal.

Once we know that the efficiency locus is the diagonal, however, it then follows that the PPF is a straight line. Because with constant returns to scale, the output of X is proportional to the distance from O_x, output of Y is proportional to the distance from O_y, and each of these distances is the length of the diagonal itself minus the other.

Formally, using the notation in the diagram below, $X = (O_x A / O_x O_y)\overline{X}$ *and thus*

$$Y = (AO_y / O_x O_y)\overline{Y} = ((O_x O_y - O_x A)/O_x O_y)\overline{Y} = \overline{Y} - (\overline{Y}/\overline{X})X$$

which is the equation of a straight line.



b. Use the Edgeworth-Bowley Box diagram to illustrate the effects on production possibilities of an increase in the endowment of capital holding the endowment of labor constant. Show how the changes in the box diagram translate to changes in the PPF.