APPENDIX

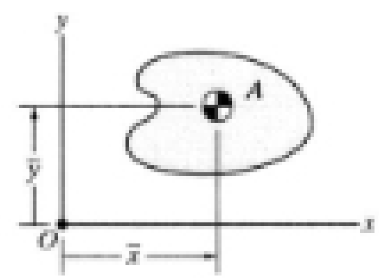
B

Properties of Areas and Lines

B.1 Areas

The coordinates of the centroid of the area A are

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA}, \qquad \bar{y} = \frac{\int_A y\,dA}{\int_A dA}.$$



The moment of inertia about the x axis I_x, the moment of inertia about the y axis I_y, and the product of inertia I_{xy} are

$$I_x = \int_A y^2\,dA, \qquad I_y = \int_A x^2\,dA, \qquad I_{xy} = \int_A xy\,dA.$$

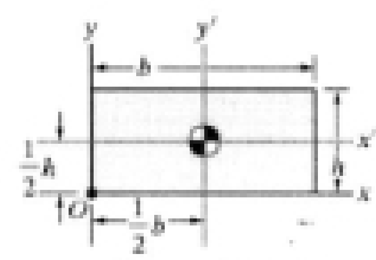
The polar moment of inertia about O is

$$J_O = \int_A r^2\,dA = \int_A (x^2 + y^2)\,dA = I_x + I_y$$

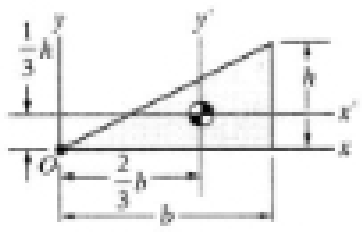
Area = bh

$$I_x = \frac{1}{3}bh^3, \qquad I_y = \frac{1}{3}hb^3, \qquad I_{xy} = \frac{1}{4}b^2h^2$$

$$I_{x'} = \frac{1}{12}bh^3, \qquad I_{y'} = \frac{1}{12}hb^3, \qquad I_{x'y'} = 0$$



Rectangular area

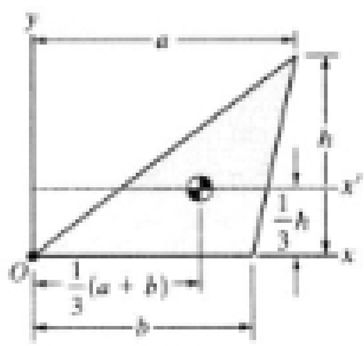


Triangular area

$$\text{Area} = \frac{1}{2}bh$$

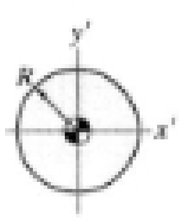
$$I_x = \frac{1}{12}bh^3, \qquad I_y = \frac{1}{4}hb^3, \qquad I_{xy} = \frac{1}{8}b^2h^2$$

$$I_{x'} = \frac{1}{36}bh^3, \qquad I_{y'} = \frac{1}{36}hb^3, \qquad I_{x'y'} = \frac{1}{72}b^2h^2$$



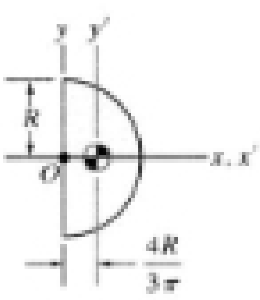
Triangular area

$$\text{Area} = \frac{1}{2}bh \qquad I_x = \frac{1}{12}bh^3, \qquad I_{x'} = \frac{1}{36}bh^3$$



Circular area

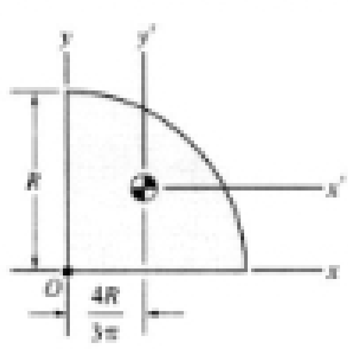
$$\text{Area} = \pi R^2 \qquad I_{x'} = I_{y'} = \frac{1}{4}\pi R^4, \qquad I_{x'y'} = 0$$



Semicircular area

$$\text{Area} = \frac{1}{2}\pi R^2 \qquad I_x = I_y = \frac{1}{8}\pi R^4, \qquad I_{xy} = 0$$

$$I_{x'} = \frac{1}{8}\pi R^4, \qquad I_{y'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4, \qquad I_{x'y'} = 0$$



Quarter-circular area

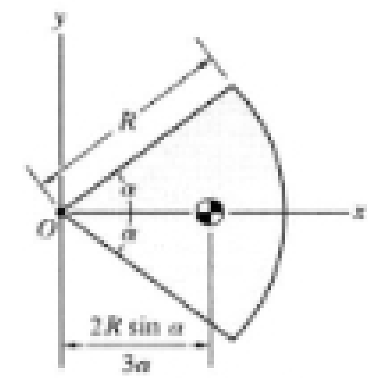
$$\text{Area} = \frac{1}{4}\pi R^2 \qquad I_x = I_y = \frac{1}{16}\pi R^4, \qquad I_{xy} = \frac{1}{8}R^4$$

$$I_{x'} = I_{y'} = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)R^4, \qquad I_{x'y'} = \left(\frac{1}{8} - \frac{4}{9\pi}\right)R^4$$

Area $= \alpha R^2$

$$I_x = \frac{1}{4}R^4\left(\alpha - \frac{1}{2}\sin 2\alpha\right), \qquad I_y = \frac{1}{4}R^4\left(\alpha + \frac{1}{2}\sin 2\alpha\right),$$

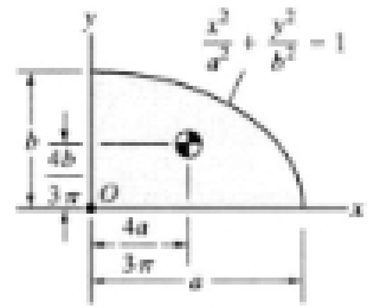
$$I_{xy} = 0$$



Circular sector

Area $= \dfrac{1}{4}\pi ab$

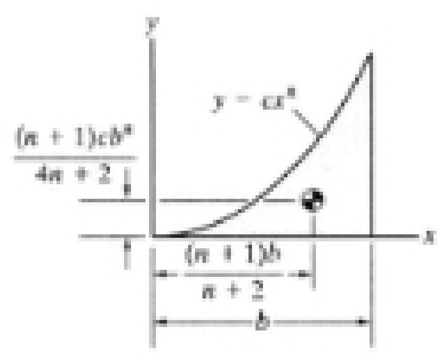
$$I_x = \frac{1}{16}\pi ab^3, \qquad I_y = \frac{1}{16}\pi a^3 b, \qquad I_{xy} = \frac{1}{8}a^2b^2$$



Quarter-elliptical area

Area $= \dfrac{cb^{n+1}}{n+1}$

$$I_x = \frac{c^3b^{3n+1}}{9n+3}, \qquad I_y = \frac{cb^{n+3}}{n+3}, \qquad I_{xy} = \frac{c^2b^{2n+2}}{4n+4}$$

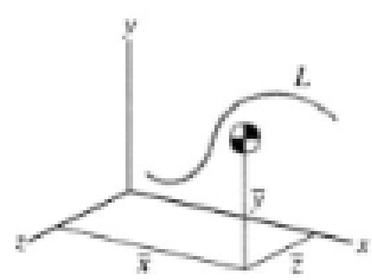


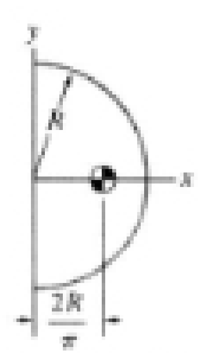
Spandrel

B.2 Lines

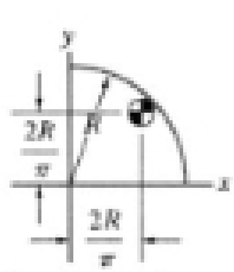
The coordinates of the centroid of the line L are

$$\bar{x} = \frac{\int_L x\, dL}{\int_L dL}, \qquad \bar{y} = \frac{\int_L y\, dL}{\int_L dL}, \qquad \bar{z} = \frac{\int_L z\, dL}{\int_L dL}.$$

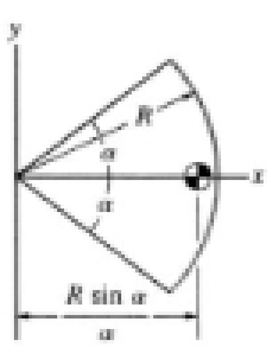




Semicircular arc



Quarter-circular arc



Circular arc