General Physics - E&M (PHY 1308) Lecture Notes

Lecture 026: The Application of Ampere's Law

SteveSekula, 1 November 2010 (created 31 October 2010)

no tags

Goals

- Demonstrate the application of Ampere's Law
- Discuss the magnetic field due to a solenoid, and its applications

Application: Solar Currents

Show the movie of the solar flares again.

Solar storms consist of large currents of electrically charged particles moving in the sun's magnetic fields. Consider a rectangular Amperian Loop that has long dimension $L = 400 \times 10^9 \text{m}$ and is in the presence of a constant magnetic field whose strength is $B = 2 \times 10^{-3} \text{T}$. What is the total current enclosed by the loop?

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

According to our picture (Wolfson Figure 26.30b), two legs of the loop lie in the direction of the magnetic field, so $\vec{B} \cdot d\vec{r} = Bdr$. For the other two, the dot product is zero. Thus:

$$\oint \vec{B} \cdot d\vec{r} = 2 \int Bdr = 2B \int dr = 2BL$$

This is then equal to:

$$2BL = \mu_0 I$$

We can then solve for the current:

$$I = 2\frac{BL}{\mu_0} = 2\,\frac{(2.0 \times 10^{-3}\mathrm{T})(400 \times 10^{9}\mathrm{m})}{(4\pi \times 10^{-7}\mathrm{N/A^2})} = 10^{12}\mathrm{A}$$

Conceptual Puzzle: Direction of Currents in Wired

Ampere's Law is also useful for solving for the direction of unknown currents in wires. Consider three parallel wires, labeled A, B, and C, in which equal magnitude current flows in the same direction in two of them but in the opposite direction in the third (you don't know which is which at first). Consider two loops, one which encircles A and B (Loop 1) and one which encircles B and C (Loop 2). If the following is true:

$$\oint \vec{B} \cdot d\vec{r} \neq 0 \text{ (Loop 2)}$$

Then:

1. What is $\oint \vec{B} \cdot d\vec{r}$ around Loop 1?

   ANSWER

   If the above is true about Loop 2, it means that the net current enclosed is NONZERO and current in B and C must flow in the same direction. That means that $\oint \vec{B} \cdot d\vec{r} = 0$ (Loop 1) because current must be opposite in one of the three wires to the flow of the other two, and that means current enclosed would be ZERO.

2. Which wire carries the current opposite the other two?

   SOLUTION

   As argued above, it must be wire A.

Application: Magnetic Field in a Long Wire

Consider an infinite straight wire along the x-axis. We've previously treated the wire as thin and calculated the magnetic field outside the wire. We found it to be:

$$B_{outside} = \frac{\mu_0 I}{2\pi y}$$

where y is the distance from the wire.

Let's consider a long, straight wire carrying a constant current, I. The wire now has a thickness; let's treat is as a cylinder whose radius is R. We recognize that this problem has *line symmetry*. This means that we expect that no matter what, the magnetic field will depend only on the radial distance from the center of the wire (the cylinder axis).

Outside the wire, we expect the magnetic field lines to circle around the wire. We can choose our Amperian loops to take advantage of this fact, and make them also circles centered on the wire axis. What about inside the wire? If we draw a circular Amperian loop inside the wire, it will enclose a fraction of the current but that current will still be a cylindrical flow with circular magnetic field lines surrounding it. This in all cases in this problem:

$$\oint \vec{B} \cdot d\vec{\ell} = \oint B d\ell = B(2\pi r)$$

where r is the radius of the enclosed cylinder.

Let's consider the two big cases in this problem: outside the wire, where the enclosed current is fixed and equal to I, and inside the loop where the enclosed current is a fraction of the total current (to be determined).

Outside the wire

Here, the enclosed current is I no matter how far we are from the wire.