Truth Table

Recall that if a sentence is a statement, then it must have well-defined truth values.

The truth table for a given statement form gives the truth values that correspond for all combinations of truth values for the component statement variables.

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

Logical Equivalence

Definition

We say that two statement forms are logically equivalent if, and only if, they have identical truth values for each possible substitution of statements for their statement variables.

If statement forms P and Q are logically equivalent then we note that $P \equiv Q$.

Example 3

1. Show that $\sim(\sim p) \equiv p$.

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

Note p and $\sim(\sim p)$ are logically equivalent because they have identical truth values.

2. Show that $\sim(p \vee q)$ and $\sim p \vee \sim q$ are not logically equivalent.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

What does this tell us?

We cannot distribute $\sim$ without changing

In the previous example we showed that $\sim (p \vee q) \not\equiv \sim p \vee \sim q$. So how do we negate the disjunction of p and q?

De Morgan's Laws

The negation of an $\wedge$ statement is logically equivalent to the $\vee$ statement in which each compound is negated.

$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

The negation of an $\vee$ statement is logically equivalent to the $\wedge$ statement in which each compound is negated.

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Example 4

Write the negations of the following.

1. Miley Cyrus is a timid, respectable performer.

p = Miley Cyrus is a timid performer
q = Miley Cyrus is a respectable performer

Statement: $p \wedge q$
Negation: $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Miley Cyrus is a performer who is not both timid and respectable.

2. It will either rain or it will be sunny tomorrow.

$\underbrace{\text{It will either rain}}_{p}$ or $\underbrace{\text{it will be sunny tomorrow}}_{q}$

statement: $p \vee q$
Negation: $\sim (p \vee q) \equiv \sim p \wedge \sim q$

It will neither rain nor be sunny tomorrow.

3. $-\pi \leq x < 2e$

$p = [x \geq -\pi]$, $q = [x < 2e]$

statement: $p \wedge q$
Negation: $\sim (p \wedge q) \equiv \sim p \vee \sim q$

$x < -\pi$ or $x \geq 2e$

Definition

A tautology is a statement form that is always true regardless of the truth values of the individual statements substituted for its statement values. It is denoted by **t**.

A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. It is denoted by **c**.

Example 5

1. Show that $p \vee \sim p$ is a tautology and that $p \wedge \sim p$ is a contradiction.

p	$\sim p$	$p \vee \sim p$ (tautology)	$p \wedge \sim p$ (contradiction)
T	F	T	F
F	T	T	F

all rows are true regardless of value of p.

truth values are always false regardless of input p.

2. Show that $p \wedge \mathbf{t} \equiv p$ and $p \wedge \mathbf{c} \equiv \mathbf{c}$.

p	$\mathbf{t}$	$\mathbf{c}$	$p \wedge \mathbf{t}$	$p \wedge \mathbf{c}$
T	T	F	T	F
F	T	F	F	F

logically equivalent because they have the same truth values

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Is propositional calculus (logic) a Boolean algebra?

YES!! see the next sheet