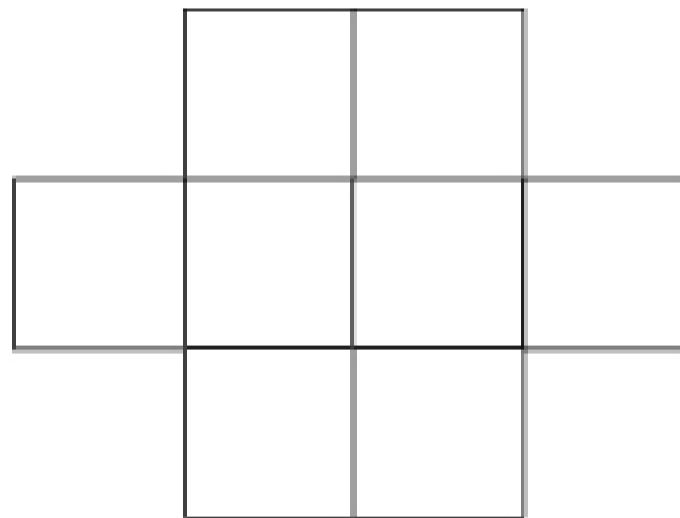


## CSP

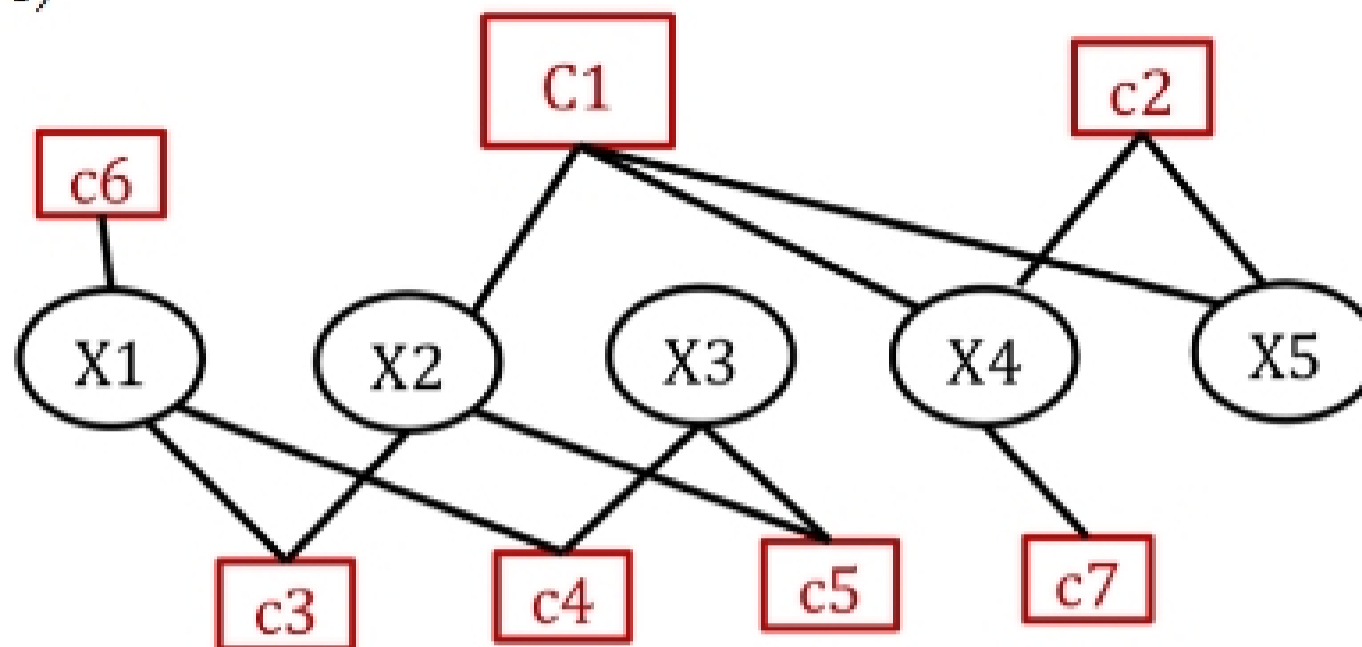
- 1) Consider the following binary constraint network: There are 4 variables:  $X_1, X_2, X_3, X_4$  with the domains:  $D_1=\{1,2,3,4\}, D_2=\{3,4,5,8,9\}, D_3=\{2,3,5,6,7,9\}, D_4=\{3,5,7,8,9\}$ . The constraints are  $X_1 \geq X_2, X_2 > X_3$  or  $X_3 - X_2 = 2, X_3 \neq X_4$ .
  - a. Draw the constraint graph.
  - b. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final network)
  - c. Is the network consistent? If yes, give a solution.
  
- 2) Consider the 8 squares positioned as follows:



The task is to label the boxes above with the numbers 1-8 such that the labels of any pair of adjacent squares (i.e. horizontal, vertical or diagonal) differ by at least 2 (i.e. 2 or more).

- a. Write all constraints and draw the constraint graph.
- b. Is the network arc-consistent? If not, compute the arc-consistent network. (show the whole process of enforcing arc-consistency and not just the final arc-consistent network)
- c. Is the network consistent? If yes, give a solution.

3)



- Variables  $X_i$  can take values  $\{0, 1, 2\}$
- Constraints  $C_1, \dots, C_5$  enforce the fact that the variables in their arguments must all have different values.
- Unary constraints  $C_6, C_7$  enforce:  
 $C_6: X_1 = 0;$   
 $C_7: X_4 \in \{1, 2\}$

In the following you must always use the “minimum remaining values heuristic” (MRV) to choose a variable. Use the “degree heuristic” (DH) as a tie-breaker. To assign a value you must use the least constraining value heuristic (LCV).

1. [2pts] Write out the domains of all variables.  
Use the unary constraints to simplify the domains of  $X_1, X_4$ .
2. [2pts] Use MRV & DH to choose the first variable and assign it a value.
3. [4pts] Use forward checking to simplify the domains of the “neighbors” of that variable.  
(neighbors are all variables that share some constraint with that variable.)
4. [4pts] Simplify the constraint graph by eliminating the first assigned variable from it.  
Draw the graph as a “standard constraint graph” (without the constraint boxes) where edges represent constraints between *pairs of nodes*.
5. [2pts] Use MRV & DH to choose the next variable and assign it a value.
6. [4pts] Make the entire graph arc-consistent. Provide a solution to the CSP.
7. [4pts] Express the time complexity of arc-consistency in terms of  $d$  (number of states) and  $n$  (number of nodes) for general graphs. Is arc-consistency exponential or polynomial in  $d, n$ ?
8. [2pts] Same as 7 but now for a tree-structured constraint graph.