ME 201/MTH 281/ME 400/CHE 400
ASSIGNMENT #1 2010

Assignments handed in by 6 PM on Wednesday September 8 will receive a 5-point bonus. Assignments handed in after that but by 6 PM on Thursday September 9 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday. Most of this assignment is review material from MTH 163 (or MTH 165) and MTH 164.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
1.1 Continuous Systems and Partial Differential Equations	W Sept. 1	1.1
1.2 Heat Conduction and the Diffusion Equation	Th Sept. 2	1.2-1.5

HOMEWORK PROBLEMS
REVIEW PROBLEMS FROM MTH 164

In problems (1) – (3) let **F** and **G** be the vector fields given by $\mathbf{F} = 3y^2\mathbf{i} - 2x^2\mathbf{j} + z^2\mathbf{k}$ and $\mathbf{G} = (2x - y + 3y^2)\mathbf{i} + (-x - 2x^2 - 4y + z)\mathbf{j} + (y + 2z + z^2)\mathbf{k}$.

(1) (10 points)
(a) (5 points) Calculate $\nabla\cdot\mathbf{F}$ and $\nabla\cdot\mathbf{G}$.
(b) (5 points) Let S be the surface of the ellipsoid $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$. Here a, b, and c are positive constants. Let **n** be the unit exterior normal to the surface and $d\sigma$ the element of surface area on S. Show that $\oiint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \oiint_S \mathbf{G}\cdot\mathbf{n}\,d\sigma$.

(2) (15 points)
(a) (5 points) Calculate $\nabla\times\mathbf{F}$ and $\nabla\times\mathbf{G}$.
(b) (5 points) It is known from vector calculus that if a vector field **H** has a zero curl, then there exists a scalar field Φ such that $\mathbf{H} = \nabla\Phi$. Use this to show that $\mathbf{F} = \mathbf{G} + \nabla\Phi$.
(c) (5 points) Find Φ explicitly.

(3) (15 points) Let C be any curve starting at point P_1 with coordinates (-1, -1, -1), and ending at point P_2 with coordinates (1, 1, 1). Show that $\int_C \mathbf{F}\cdot d\mathbf{s} = \int_C \mathbf{G}\cdot d\mathbf{s}$, where $d\mathbf{s}$ is the element of vector arclength along C.

(4) (10 points) Let the temperature distribution in the air be $T(x,y,z) = T_0 + ax + by - cz$, where x, y, and z, are rectangular coordinates, with positive z being upward, positive x East and positive y North. The constants are $T_0 = 20\,^\circ\mathrm{C}$, a=0.2 ˚C/km, b = 0.3 ˚C/km, and c = 0.4 ˚C/km.
A small plane is flying at constant speed $V_0 = 300$ km/hr. The ground heading of the plane is northeast, and the plane is climbing at a constant angle α. The external thermometer on the plane shows that the temperature is increasing at a rate 1.47 °C/minute. What is the angle of climb of the plane? At this speed and direction what would the angle of climb have to be for the measured temperature to remain constant?

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REVIEW PROBLEMS FROM MTH 163 OR 165

(5) (10 points) Solve the initial-value problem $\frac{dx}{dt} + 2tx = 2te^{-t^2}$, $x(0) = 3$.

(6) (10 points) Solve the initial-value problem $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 20x = 0$, $x(0) = 1$, $\frac{dx}{dt}(0) = 2$.

(7) (10 points) Solve the initial-value problem $\frac{d^2y}{dx^2} + 9y = 0$, $y(0) = 0$, $\frac{dy}{dx}(0) = 6$.

(8) (10 points) Solve the initial-value problem $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$, $y(0) = 0$, $\frac{dy}{dx}(0) = 2$.

(9) (10 points) Find the general solution of $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = 0$.

CHALLENGE PROBLEM

Under certain conditions which we will discuss later in the course, a vector field is characterized by its divergence and its curl. That raises the question of whether there are any non-constant vector fields with zero divergence and zero curl. You are asked to answer that question here by finding an explicit example of such a field. (Hint: See the vector calculus result stated in problem 2.)