

ME 406 ASSIGNMENT #1

PROBLEMS DUE IN CLASS THURSDAY JANUARY 22, 2009

This assignment is not typical of the assignments in this course. It has one specific purpose: to give you a crash course in Mathematica. Mathematica is the tool that we use in this course and it is important for you to become familiar with it as rapidly as possible.

PROBLEMS

Work through the entire Mathematica tutorial (available on the course CD). You do not need to hand in anything from the tutorial, but the problems below will require techniques that are illustrated in the tutorial.

(1) (20 points) Consider the function $f(x) = e^x - 4\sin(x)$.

(a) Plot this function from $x = -5$ to $x = 5$. Label your axes in the plot.

(b) Use FindRoot to find the two positive roots of this function.

(2) (20 points) Consider the ellipse given by $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$. Give a parametric representation of this ellipse and use ParametricPlot to generate a plot of it.

(3) (35 points) Consider the initial-value problem given below for $x(t)$. The equation satisfied by x is the equation for a free damped linear oscillator.

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0 \quad , \quad x(0) = a \quad , \quad \frac{dx}{dt}(0) = b \quad .$$

In the calculations requested below, use the following parameter values:

$$m = 2 \text{ kg}, \quad c = 0.8 \text{ N}\cdot\text{s/m}, \quad k = 8 \text{ N/m}, \quad a = 1 \text{ m}, \quad b = 2 \text{ m/s} .$$

(a) Use DSolve in Mathematica to find an analytical solution of the problem.

(b) Use Mathematica to check your solution. Check that the equation is satisfied and check the two initial conditions.

(c) Use the command Plot in Mathematica to plot $x(t)$ versus t .

(d) Use the command Plot in Mathematica to plot dx/dt versus t .

(e) Use the command Show in Mathematica to combine your plots of parts (c) and (d) into a single plot.

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(3) (continued)

(f) Use `NDSolve` to find a numerical solution of the problem, and plot $x(t)$ versus t from your numerical solution.

(g) Convert the second order equation to a system of two first order equations, and use `NDSolve` to find a numerical solution of the system. Plot $x(t)$ versus t .

(4) (25 points) Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 & 0 \\ -2 & 0 & 3 & 2 \\ 3 & 3 & 5 & -6 \\ 0 & 2 & -6 & 1 \end{pmatrix}.$$

(a) Use Mathematica to find the inverse of \mathbf{A} . Check (using Mathematica) that the inverse times \mathbf{A} gives the identity matrix. Use Mathematica to solve the linear equations $\mathbf{A}\mathbf{X} = \mathbf{b}$, where

$$\mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Use Mathematica to check your solution.

(b) Use the commands `Eigenvalues` and `Eigenvectors` to find the eigenvalues and eigenvectors of \mathbf{A} . Because \mathbf{A} is symmetric and the eigenvalues are distinct, the eigenvectors should be pairwise orthogonal. Use Mathematica to verify this.