

PRINT YOUR NAME : _____

From the questions below, please choose and solve 10 problems only.

The homework is worth 10 points.

Show all of your work and put a box around your final answer.

Number each attempted question clearly.

Write legibly (that is, suitably large and suitably dark); if the grader can't read your answer, it's consider uncompleted.

Question 1 Let $f(x) = x^2$. For each of the following functions, (i) state how the graph is related to the graph of f , and (ii) use this information to draw the graph.

(a) $g(x) = f(x + 1)$

(b) $h(x) = f(x) + 1$

(c) $l(x) = f(5x)$

(d) $k(x) = 5f(x)$

Question 2 Determine the domain of the function.

(a) $g(t) = \frac{\sqrt{t}}{t^2 - 9}$

(b) $h(x) = \frac{x + x^{-1}}{(x - 3)(x + 4)}$

Question 3 Linear vs exponential data. For each table of data, examine the data and then take a guess as to whether the data would best be modeled by a linear or exponential function. Once you make your choice, see if you can write a function that models the data.

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Part A

x	0	1	2	3
y	-3	-1.5	0	1.5

Part B

x	0	1	2	3	4
y	16	4	1	1/4	1/16

Part C

x	0	1	2	3
y	3	4.5	6.75	10.125

Part D

x	0	1	2	3	4
y	16	12	8	4	0

Question 4 Roots of quadratic functions. For which value(s) of b does the function $f(x) = x^2 + bx + 1$ have

(a) two real roots?

(b) only one real root?

(c) no real roots?

Question 5 Piecewise graphs. Graph each of the given functions. Be sure to clearly indicate possible points that may be significant. 2

(a) $f(x) = x^2 + 1$

$$(b) g(x) = -\frac{x}{2} + 6$$

$$(c) h(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ -\frac{x}{2} + 6, & x > 2 \end{cases}$$

$$(d) i(x) = \begin{cases} x^2, & x \leq 2 \\ -\frac{x}{2} + 6, & x > 2 \end{cases}$$

Question 6 **Function composition.** Use the functions in Question 5 to answer the following:

- (a) Find $(f \circ g)(x)$ and simplify your answer.
- (b) Find $(g \circ f)(x)$ and simplify your answer.

Question 7 Let $f(t)$ be the population (in thousands of people) of Math City, as a function the time t (in years), where $t = 0$ corresponds to the year 2020. In 2020, the population was 34000, and each year the population increases by 3000 people.

- (a) Model the population by finding a formula for $f(t)$.
- (b) Evaluate the expression $f(6)$, and write a sentence explaining what this means.
- (c) Solve the equation $f(t) = 64$, and write a sentence explaining what this means.

Question 8 Let $L = f(h)$ be the amount of water (in liters) a species of tree needs per week when it is h meters tall. A particular tree of this species in the PSU arboretum is h_P meters tall, and so it requires $f(h_P)$ liters of water per week, and we define $v_P = f(h_P)$.

- (a) What is the practical meaning of the expression $f(h_P + 3)$?
- (b) What is the practical meaning of the statement $f(h_P + 1) = v_P + 3$?
- (c) What is the practical meaning of $f(h_P/2)$?
- (d) What is the practical meaning of $f(h_P - 5) = v_P/2$?

Question 9 A spherical balloon with radius r inches has a volume $V(r) = \frac{4}{3}\pi r^3$.

- (a) Find an expression for the amount of air required to inflate the balloon so that the radius increases from r to $r + 3$.
- (b) Find an expression for the amount of air required to double the radius of the balloon from a radius of r to a radius of $2r$.
- (c) If the balloon starts with a volume of 36π cubic inches and the radius is increasing at 1 inch per minute, then how long will it take for the balloon to inflate to a volume of 200 cubic inches?

Question 10 For each of the following scenarios, determine what type of function — linear or power — fits the given description, and find a formula for $y = f(x)$. Assume for all the situations that $y = 1$ when $x = 1$.

- (a) Each time x increases by a factor of 2, y increases by a factor of 4.
- (b) Each time x increases by a factor of 2, y increases by a factor of 8.
- (c) Each time x increases by 1, y increases by 3.
- (d) Each time x increases by 1, y decreases by 3.
- (e) Each time x increases by a factor of 2, y decreases by a factor of 2.
- (f) Each time x increases by a factor of 2, y decreases by a factor of 4.

Question 11 Let $g(t)$ be the population (in thousands of people) of Calculus City, as a function the time t (in years), where $t = 0$ corresponds to the year 2020. In 2020, the population was 10000, and each year the population increases by 30%.

- (a) Model the population by finding a formula for $g(t)$, using the appropriate choice of a linear, polynomial, power, trigonometric, or exponential function, or a piecewise combination thereof.
- (b) Evaluate the expression $g(6)$, and write a sentence explaining what this means.
- (c) Solve the equation $g(t) = 16.9$, and write a sentence explaining what this means.

Question 12 Let $h(t)$ be the population (in thousands of people) of Algebra City, as a function the time t (in years), where $t = 0$ corresponds to the year 2020. In 2020, the population was 20000. Between 2020 and 2030, the population grows by 5000 people per year, but then after 2030 the population declines by 8% per year.

- (a) Model the population by finding a formula for $h(t)$, using the appropriate choice of a linear, polynomial, power, trigonometric, or exponential function, or a piecewise combination thereof.
- (b) Evaluate the expression $h(6)$, and write a sentence explaining what this means.
- (c) Evaluate the expression $h(12)$, and write a sentence explaining what this means.

Question 13 Population A is 20 thousand at $t = 1$, where t is in years. It grows by 15% per year.

- (a) Find a formula for $A(t)$, the population (in thousands) at time t .
- (b) What is the doubling time? (The **doubling time** is the time it takes an exponentially growing function to double. It's analogous to the half-life of an exponentially decaying function.)

Question 14 Answer the following parts.

- (a) Write down a formula for the amount of money you would have in your account after t years if it started at \$1000 and the account pays 2.5% annual interest compounded monthly.
- (b) If you buy a car for \$20,000 and it loses half its value every year when will the car be worth \$3,000?
- (c) A population is known to grow exponentially. If it starts at 100 and is 150 after 3 months when will the population be 300?