

MATH 285 E1/F1 GRADED HOMEWORK SET 2
DUE FRIDAY SEPTEMBER 26 IN LECTURE

LET'S CHANGE IT UP WITH SOME FRESH NEW INSTRUCTIONS: This time, the homework has **two parts, A and B**. Please turn in each part separately, with your **name and section** clearly marked on each part. Please staple all the pages for a particular part together, but do not staple the two parts to each other. When you turn the homework in, there will be two boxes. *Thank you!*

PART A

- (1) (5 points) Consider the equation

$$(2y + xe^{xy}) \frac{dy}{dx} + x + ye^{xy} = 0$$

Show that this equation is exact, and find an implicit equation for the solution.

- (2) (5 points) Let $P(t)$ denote the number of technology startups in the San Francisco Bay area, where time t is measured in months. Startups are created when two recent graduates decide to found one, and they either fail or they are bought by Google. The rate at which startups are formed is given by $(10 - 0.01P)P$ per month. Also, each month, 10% of the startups fail, and 5% are bought by Google. How many startups do you expect to exist at a particular time many months into the future? That is, what is $\lim_{t \rightarrow \infty} P(t)$?

PART B

- (3) (5 points) Suppose that $y_1(x)$ and $y_2(x)$ are two solutions of the *nonhomogeneous* equation

$$A(x)y'' + B(x)y' + C(x)y = F(x).$$

Prove that their difference $y_1(x) - y_2(x)$ is a solution of the *homogeneous* equation

$$A(x)y'' + B(x)y' + C(x)y = 0.$$

- (4) (15 points)
(a) Find the general solution of the second order linear homogeneous equation

$$4y'' + 8y' + 3y = 0.$$

- (b) Using your innate cleverness, find a particular solution to the nonhomogeneous equation

$$4y'' + 8y' + 3y = 15.$$

- (c) Using the results of the previous two parts, find solution of the initial value problem

$$4y'' + 8y' + 3y = 15, \quad y(0) = 0, \quad y'(0) = 0.$$