PRINT YOUR NAME : _____

From the questions below, please choose and solve 5 problems only.
The homework is worth 10 points. Each question is worth 2 points.
Show all of your work and put a box around your final answer.
Number each attempted question clearly.
Write legibly (that is, suitably large and suitably dark); if the grader can't read your answer, it's consider uncompleted.

Question 1 For each part below, calculate the limit or determine that it does not exist. If a limit is ∞ or $-\infty$, say so. **Show all necessary steps.**

(a) $\lim_{x \to 6} \dfrac{x^2 - 36}{x + 6}$

(b) $\lim_{x \to 6} \dfrac{x - 36}{(x - 6)^2}$

(c) $\lim_{x \to 6} \dfrac{x^2 - 36}{x - 6}$

Question 2 Assume that $\lim_{x \to 2} f(x) = 2$. For each part below, calculate the limit or determine that it does not exist. If a limit is ∞ or $-\infty$, say so. **Show all necessary steps.**

(a) $\lim_{x \to 2} \big(f(x)\big)^4$

(b) $\lim_{x \to 2} \dfrac{1}{\big(f(x)\big)^2}$

(c) $\lim_{x \to 2} \big(3x + x^2 f(x)\big)$

(d) $\lim_{x \to 2^+} \dfrac{f(x)}{x - 2}$

(e) $\lim_{x \to 2^-} \dfrac{f(x)}{x - 2}$

(f) $\lim_{x \to 2} \dfrac{f(x)}{x - 2}$

Question 3 For each of the following, draw the graph of a function on $[0, 5]$ with the given properties.

(a) f is not continuous at $x = 1$ but $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$ exist and are equal.

(b) f has a jump discontinuity at $x = 2$, and $\lim_{x \to 2^+} f(x) = f(2)$.

(c) f has a removable discontinuity at $x = 1$, a jump discontinuity at $x = 2$, and

$$\lim_{x \to 3^-} f(x) = -\infty, \qquad \lim_{x \to 3^+} f(x) = 2.$$

Question 4 Consider the function

$$f(x) = \begin{cases} x^2 + 3, & x < 1, \\ 10 - x, & 1 \le x < 2, \\ 11, & x = 2, \\ 6x - x^2, & x > 2. \end{cases}$$

(a) For which value(s) of a does $\lim_{x \to a} f(x)$ NOT exist?

(b) For which value(s) of b is $f(x)$ NOT continuous at $x = b$?

(c) Draw the graph of f.

Question 5 For each of the given functions, find the value of the constant c which would make the function continuous for all values of x.

(a) $\begin{cases} x^2 - c, & x < 5 \\ 4x + 2c, & x \geq 5 \end{cases}$

(b) $\begin{cases} \dfrac{5\sin(x)}{x}, & x < 0 \\ e^x + \sqrt{x^2 + c}, & x \geq 0 \end{cases}$

Question 6 Evaluate each limit. Write all of your steps and show all necessary details.

(a) $\lim\limits_{x \to 1^+} \sqrt{x-1}\cos\left(\dfrac{1}{x-1}\right)$

(b) $\lim\limits_{x \to \pi/2} x\cos(x)$

(c) $\lim\limits_{x \to \infty} \sin\left(\dfrac{\pi}{x}\right)$

(d) $\lim\limits_{x \to 2} \ln(5x^2 - 7)$

(e) $\lim\limits_{x \to \infty} \ln(5x^2 - 7)$

(f) $\lim\limits_{x \to 2} \left[\ln(5x^2 - 7) - \ln(3x^2 + 2)\right]$

(g) $\lim\limits_{x \to \infty} \left[\ln(5x^2 - 7) - \ln(3x^2 + 2)\right]$

(h) $\lim\limits_{x \to 0^+} \sqrt{x}\left(1 + \sin^2\left(\dfrac{\pi}{x}\right)\right)$

Question 7 According to the Michaelis-Menten equation, when an enzyme is combined with a substrate of concentration s (in millimolars), the reaction rate (in micromolars/min) is

$$R(s) = \frac{As}{K + s}$$

for constants A and K.

(a) Show, by computing $\lim\limits_{s \to \infty} R(s)$, that A is the limiting reaction rate as the substrate concentration increases without bound.

(b) Show that the reaction rate attains one-half of the limiting value when the substrate concentration is K millimolars.

Question 8 Ecologists predict the population of white-tailed deer in the state of Kentucky t years after 2004 will be

$$P(t) = \frac{1,072,764e^{0.2311t}}{0.19196 + e^{0.2311t}}.$$

According to the ecologists' model, what is the long term population of white-tailed deer?

Question 9 After an oil spill in the Gulf of Mexico, it is estimated that the concentration of oil in the water will total $C(t) = 20 + 85e^{-\frac{t}{10}}$ mg per liter, where t is the number of years after the spill.

(a) A safe concentration for the environment is considered to be 30 mg per liter. How long will it take before the concentration levels reach this amount?

(b) What is the concentration of oil in the water expected to be in the long term?

Question 10 Because the form $\frac{\infty}{\infty}$ is indeterminate, knowing that functions f and g satisfy

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$$

tells us very little about $\lim_{x\to\infty} \frac{f(x)}{g(x)}$.

(a) Find formulas for f and g such that $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is an $\frac{\infty}{\infty}$ form that has a limit of ∞.

(b) Find formulas for f and g such that $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is an $\frac{\infty}{\infty}$ form that has a limit of 0.

(c) Find formulas for f and g such that $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is an $\frac{\infty}{\infty}$ form that has a limit of 1.

(d) Find formulas for f and g such that $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is an $\frac{\infty}{\infty}$ form that has a limit of 13.72.

Hint: you can answer each part by choosing f and g to be appropriate polynomials.

Question 11 Because the form $0 \cdot \infty$ is indeterminate, knowing that functions f and g satisfy

$$\lim_{x\to\infty} f(x) = 0, \qquad \lim_{x\to\infty} g(x) = \infty$$

tells us very little about $\lim_{x\to\infty} f(x)g(x)$.

(a) Find formulas for f and g such that $\lim_{x\to\infty} f(x)g(x)$ is a $0 \cdot \infty$ form that has a limit of ∞.

(b) Find formulas for f and g such that $\lim_{x\to\infty} f(x)g(x)$ is a $0 \cdot \infty$ form that has a limit of 0.

(c) Find formulas for f and g such that $\lim_{x\to\infty} f(x)g(x)$ is a $0 \cdot \infty$ form that has a limit of 1.

(d) Find formulas for f and g such that $\lim_{x\to\infty} f(x)g(x)$ is a $0 \cdot \infty$ form that has a limit of 13.72.

Hint: you can answer each part by choosing $f(x)$ and $g(x)$ to each be of the form Cx^p for appropriate constants C and p.