MATH 285 E1/F1 GRADED HOMEWORK SET 6
DUE FRIDAY NOVEMBER 21 IN LECTURE

This time, the homework has **just one part**. Please staple your homework together, and put your **name and section** on it. *Thank you!*

(1) (15 points) Consider the eigenvalue problem

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = 0 \\ y(1) = 0 \end{cases}$$

Find the eigenvalues and eigenfunctions for this problem. That is, find the values of λ for which the problem has a nontrivial solution, and find those nontrivial solutions. *Hint:* The smallest eigenvalue is $\pi^2 + 1$, with associated eigenfunction $e^{-x} \sin \pi x$. (In your answer you should verify this.)

(2) (10 points) Find the solution of the heat problem on the interval $0 \leq x \leq 5$:

$$\begin{cases} \frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2} \\ \frac{\partial u}{\partial x}(0,t) = 0 \\ \frac{\partial u}{\partial x}(5,t) = 0 \\ u(x,0) = \cos^2 10\pi x \end{cases}$$

(3) (15 points) Find the solution of the heat problem on the interval $0 \leq x \leq 1$:

$$\begin{cases} \frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2} \\ u(0,t) = 0 \\ u(1,t) = 1 \\ u(x,0) = x^2 \end{cases}$$

You should use the following strategy:

(a) Find a steady-state solution u_0 of all parts of the problem except the initial condition. That is, find a function $u_0(x,t)$ that is constant in time ($\frac{\partial u}{\partial t} = 0$), that satisfies the heat equation, and that satisfies the conditions $u_0(0,t) = 0$ and $u_0(1,t) = 1$.

(b) Posit $w = u - u_0$, and show that w must solve a slighty different heat problem:

$$\begin{cases} \frac{\partial w}{\partial t} = 5\frac{\partial^2 w}{\partial x^2} \\ w(0,t) = 0 \\ w(1,t) = 0 \\ w(x,0) = x^2 - u_0(x,0) \end{cases}$$

(w is called the transient term).

(c) Using the methods described in the lectures, find the solution $w(x,t)$, and hence $u(x,t)$.