From the questions below, please choose and solve any 5 problems only.

The homework is worth 10 points. Each question is worth 2 points.

Show all of your work and put a box around your final answer.

Number each attempted question clearly.

Write legibly (that is, suitably large and suitably dark); if the grader can't read your answer, it's consider uncompleted.

Question 1 Find the derivative of each function.

(a) $f(x) = \ln\left(\frac{x+1}{x^3+1}\right)$

(b) $g(x) = \frac{x(x^2+1)}{\sqrt{x+1}}$

(c) $h(x) = x^{x^2}$

Question 2 The function $M(t) = (a + (b-a)e^{kmt})^{1/m}$ is known as the von Bertalanffy function and was introduced in the 1930s by Austrian biologist Karl Ludwig von Bertalanffy. Calculate $M'(0)$ in terms of the constants a, b, k and m.

Question 3 The energy (measured in ergs) associated with an earthquake of moment magnitude M_w are *related* by $\log_{10}(E) = 16.1 + 1.5M_w$. Calculate dE/dM_w for $M = 2,\ 5,\ 8$.

Question 4 A particle moves counterclockwise around the ellipse with equation $9x^2 + 16y^2 = 25$.

(a) In which of the four quadrants is $dx/dt > 0$? Explain.

(b) Find a relation between dx/dt and dy/dt.

(c) At what rate is the x-coordinate changing when the particle passes the point $(1, 1)$ if its y-coordinate is increasing at a rate of 6 m/s?

Question 5 Let $L = f(h)$ be the amount of water (in liters) a species of tree needs per week when it is h centimeters tall. Some values are given in the table below.

h	100	150	200	250	300
$f(h)$	20	30	42	58	75

Let $h = g(t) = 100 + 0.1t^2$ be the height (in centimeters) of a particular tree of this species t months after January 1, 2020. (Assume this model is valid for the first 6 years of the tree's life.)

(a) Estimate $f'(250)$. Give units. Explain the practical meaning in a sentence.

(b) Calculate $g'(39)$. Give units. Explain the practical meaning in a sentence.

(c) Estimate $\left(\frac{d}{dt} f(g(t))\right)\Big|_{t=39}$. Give units. Explain the practical meaning in a sentence.

Question 6 A parcel of air rising quickly in the atmosphere will decrease in temperature and increase in volume if it does not exchange heat with the surrounding air. For sufficiently dry air, the relationship between temperature and volume is given by $TV^{0.4} = C$ for a constant C, temperature T in Kelvin, and volume V in cubic meters. Let time t be in hours.

(a) Find $\frac{dT}{dV}$ and explain what it represents. Be sure to include units.

(b) Find $\frac{dV}{dT}$ and explain what it represents. Be sure to include units.

(c) Find $\frac{dT}{dt}$ assuming that both T and V are functions of time, and explain what it represents. Be sure to include units.

(d) Find $\frac{dT}{dt}$ if $V = 10$ m^3, $T = 295$ K and volume is increasing at a rate of 1m^3 every hour.

(e) Find $\frac{dV}{dt}$ assuming that both T and V are functions of time, and explain what it represents. Be sure to include units.

(f) Find $\frac{dV}{dt}$ if $V = 10$ m^3, $T = 295$ K and temperature is increasing at a rate of 2 K every hour.

Question 7 According to the ideal gas law pressure, P (in pascals) , volume, V (in cubic meters), and temperature, T (in kelvins) are related by the equation

$$PV = nRT$$

where R is the ideal gas constant and n is the number of moles of the gas present.

(a) Find $\frac{dP}{dt}$ assuming that both volume and temperature are changing in time.

(b) Create a model for temperature assuming that it varies sinusoidally in time starting with a minimum temperature of 300 K at the initial time of $t = 0$ hours and is at its maximum of 320 K at $t = 12$ hours.

(c) Create a model for volume assuming that starts at 10 cubic meters at $t = 0$ hours and increases by 10% every six hours.

(d) Find $\left.\frac{dP}{dt}\right|_{t=18}$ under the assumptions of parts (a), (b) and (c). Your answer will contain n and R.

Question 8 The *relative rate of change* of a function f is given by the ratio $\frac{f'}{f}$. The relative rate of change puts the additive rate of change into perspective compared to the current value of the function. After all, adding 100 people per year is much more significant for a town of hundreds than for a city of millions.

(a) Compute the relative rate of change for a power function, that is, a function of the form $f(x) = Ax^n$ for positive constants A and n. Analyze the relative rate of change as $x \to \infty$, and interpret the result.

(b) Compute the relative rate of change for an exponential function of the form $f(x) = Ae^{kx}$ for positive constants A and k. Analyze the relative rate of change as $x \to \infty$, and interpret the result.

Question 9 The volume of a sphere of radius r is $V = (4/3)\pi r^3$. If the radius is expanding at a rate of 14 inches per minute, at what rate is the volume changing when $r = 8$ in.?

Question 10 Sonya and Isaac are in motorboats located at the center of a lake. At time $t = 0$, Sonya begins traveling south at a speed of 32 mph. At the same time, Isaac takes off heading east at a speed of 27 mph.

(a) How far have Sonya and Issac traveled after 12 min?

(b) At what rate is the distance between Sony and Issac changing after 12 min?

Question 11 A traffic patrol helicopter is stationary a quarter of a mile directly above a highway. Its radar detects a car whose line-of-sight distance from the helicopter is half a mile and is increasing at the rate of 57 mph. Is the car exceeding the highway's speed limit of 60 mph? Justify your answer completely.

Question 12 Consider an electrical circuit. The voltage V (volts), current I (amperes), and resistance R (ohms) are related by Ohm's law, which states $V = IR$. Suppose that V is increasing at the rate of 2 volt/sec while I is decreasing at the rate of $\frac{1}{4}$ amp/sec.

(a) Find the equation that relates $\frac{dR}{dt}$ to $\frac{dV}{dt}$ and $\frac{dI}{dt}$.

(b) Find the rate at which R is changing when $V = 12$ volts and I =2 amps. Be sure to include units. Is R increasing or decreasing?