CMSC 250 – Jerry Alan Fails
Due: Tuesday, July 3, 2007

HW #8

Student: _____
ID #: _____

You must work alone on your homework, and homework must be **written legibly**, single-sided on your own lined paper, **or typed**, with the answers clearly labeled and in the sequential order as assigned. You must write your name and university ID number in the upper right-hand corner of your homework. Staple all pages together and be sure that your name appears on every sheet.

1. (-10 points if wrong) Write your name clearly on each page. Write the time and place of Exam 2.

2. (10 points) Suppose P(n) is a property such that
   i) P(0)
   ii) For all integers $k \geq 0$, if P(k) is true, then P(3k) is true.
   Must it follow that P(n) is true for all integers $n \geq 0$? If yes, explain why; if no, give a counterexample.

3. (70 points) For each of the following say whether it is true or false. If true, use mathematical induction to prove it. If false, give a specific counterexample.

   (a) $\forall n \in \mathbb{Z}^{\geq 1},\ 8 \mid (3^{2n} - 1)$.

   (b) $\forall n \in \mathbb{Z}^{+},\ \sum_{i=1}^{n} \left(\frac{i}{i+1} - \frac{i+1}{i+2} \right) = \left(\frac{1}{2} - \frac{n+1}{n+2} \right)$.

   (c) $\forall n \in \mathbb{Z}^{\geq 2},\ 4 \mid (3^{n} - 1)$.

   (d) $\forall n \in \mathbb{Z}^{+},\ \sum_{k=2}^{n} \frac{k}{k-1} = \left(\frac{3n}{2} - 1 \right)$.

   (e) $\forall n \in \mathbb{Z}^{\geq 6},\ \frac{n^2}{2} \geq 2n + 5$.

   (f) $\exists m \in \mathbb{Z},\ \forall n \in \mathbb{Z}^{\geq m},\ n^3 \leq 2^n$.

4. (20 points) Suppose $a_4, a_5, a_6, a_7, \ldots$ is a sequence defined as follows:

$$\begin{cases} a_1 = 0, a_2 = 1, a_3 = 2 \\ a_k = a_{k-1} + a_{k-2} + a_{k-3} \quad \text{for all integers } k \geq 4 \end{cases}$$

   (a) (5 points) What are a_4, a_5, a_6, a_7?
   (b) (5 points) What can you say about the parity (i.e. evenness and oddness) of a_k, in general?
   (c) (10 points) Prove it.

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5. (**No points will be awarded for this assignment unless this is done**) Sign your name to the following honor code statement: "I pledge on my honor that I have not given or received any unauthorized assistance on this assignment".