

## 29:194 Homework #5

Due at the beginning of class, Thursday, September 30, 2010.

1. Show that for an electric field of the form

$$\mathbf{E}(\mathbf{x}, \tau, t) = \mathbf{E}_0(\mathbf{x}, \tau) \cos(\omega t - \mathbf{k} \cdot \mathbf{x})$$

the magnetic field is given by

$$\mathbf{B}(\mathbf{x}, \tau, t) = -\frac{1}{\omega} \{[\nabla \times \mathbf{E}_0(\mathbf{x}, \tau)] \sin(\omega t - \mathbf{k} \cdot \mathbf{x}) - [\mathbf{k} \times \mathbf{E}_0(\mathbf{x}, \tau)] \cos(\omega t - \mathbf{k} \cdot \mathbf{x})\}$$

2. A mirror machine has a mirror ratio  $R_m = 2$ . A group of electrons with an isotropic velocity distribution (Maxwellian) is released at the center of the machine. In the absence of collisions, what fraction of these electrons is confined?
3. A singly ionized, 20 eV Argon plasma is confined in a magnetic cusp field such that throughout the volume of the plasma, except for the edges, the magnetic field is zero. The plasma diameter is 1.0 m, and its density is  $n = 10^{11} \text{ m}^{-3}$ . A small, 0.5 cm diameter spherical probe is placed at the center of the plasma and set to a potential of 100 V above the wall potential. At what distance from the probe surface will the measured potential be 1 V?
4. An electron of charge  $q_e = -e$  and mass  $m_e$  and an proton of charge  $q_p = e$  and mass  $m_i = m_p$  are initially at rest at  $\mathbf{x} = (0, 0, 0)$  in a magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{z}}$ . An electric field is then turned on at  $t = 0$  and increased linearly until time  $t_1 = \frac{20\pi m_i}{eB_0}$ , at which point the electric field is held constant,

$$\mathbf{E}(t) = \begin{cases} 0 & t < 0 \\ E_0(t/t_1)\hat{\mathbf{y}} & 0 \leq t \leq t_1 \\ E_0\hat{\mathbf{y}} & t > t_1 \end{cases}$$

Find the total current density as a function of time  $\mathbf{j}(t)$  due to the drifts of the two particles (neglect the current due to the fast Larmor oscillation).

5. NUMERICAL: Polarization Drift

Use the same Matlab m-files `lorentz.m`, `magnetic.m`, `electric.m`, `euler1.m`, `leapfrog2.m`, and `spm.m` as used in HW#4 and the adaptive RK45 method with a specified tolerance `RelTol= 1.10 × 10-5`. Specify  $\mathbf{B} = (0, 0, 1)$ ,  $q_i/m_i = 1$ ,  $q_e = -q_i$  and an artificial mass ratio  $m_i/m_e = 10$ . Specify an electric field that increases with time  $\mathbf{E}(t) = E_0 t/t_f \hat{\mathbf{y}}$  with  $E_0 = 0.5$  and  $t_f$  equal to 10 ion cyclotron periods. NOTE: The `hold on` command can be used to plot a second trace on the same plot; `hold off` turns this off.

- (a) Plot the trajectories over  $t = [0, t_f]$ , on the same plot, of both the ion and electron each with an initial position  $\mathbf{x}_0 = (0, 0, 0)$  and initial velocity  $\mathbf{v}_0 = (-1, 0, 0)$ .
- (b) Why do we not use a realistic mass ratio (for protons) of  $m_i/m_e = 1836$  to do this calculation? HINT: Try using  $m_i/m_e = 40$ .

6. Laser Trapping: A charged particle can be trapped by a spatially varying intense laser field. Using interference of several lasers, the electric field near a charged particle is given by

$$\mathbf{E}(\mathbf{x}, t) = E_0[1 + (x/x_0)^2] \sin(\omega t - k_y y) \hat{\mathbf{x}}.$$

Calculate the velocity of the oscillation center  $U$  as a function of position  $x$  for a particle initially at rest at  $t = 0$  at position  $\mathbf{x} = (x_0, 0, 0)$ . You may assume that the particle velocity  $v$  and laser frequency  $\omega$  satisfy  $v \ll \omega/k_y$  and  $v/x_0 \ll \omega$ .

7. NUMERICAL: Ponderomotive Force

Use the adaptive RK45 method with a specified tolerance  $\text{RelTol} = 1.0 \times 10^{-5}$ . Plot the  $x$  position of the particle vs. time  $t$  over a time  $t = [0, 50]$  for the problem above using  $E_0 = 2$ ,  $x_0 = 1$ ,  $\omega = 10$ ,  $k_y = 0.01$ , and  $q/m = 1$ .