

4.37 Find phasor form

$$a. v(t) = \underbrace{155}_{\text{magnitude}} \cos(377t - \underbrace{25^\circ}_{\text{angle}}) \text{ V}$$

$$V(j\omega) = 155 \angle -25^\circ \text{ V}$$

$$b. v(t) = 5 \sin(1,000t - 40^\circ) \text{ V}$$

$$= \underbrace{5}_{\text{mag.}} \cos(1,000t - \underbrace{130^\circ}_{\text{angle.}}) \text{ V}$$

$$V(j\omega) = 5 \angle -130^\circ \text{ V}$$

Note: I would normally suggest using radians but the problem was set up in degrees.

$\sin(\theta) = \cos(\theta - 90^\circ)$
We use $\cos()$ as our reference so we need to shift $\sin()$ by 90° to turn it into a $\cos()$.

$$c. i(t) = 10 \cos(10t + 63^\circ) + 15 \cos(10t - 42^\circ) \text{ A}$$

$$I(j\omega) = 10 \angle 63^\circ + 15 \angle -42^\circ \text{ A}$$

Convert to rectangular form for addition

$$I(j\omega) = 10 \cos(63^\circ) + j10 \sin(63^\circ) + 15 \cos(-42^\circ) + j15 \sin(-42^\circ) \text{ A}$$

$$= 4.54 + j8.91 + 11.15 - j10.04 \text{ A}$$

$$= 15.69 - j1.13 \text{ A}$$

$$= \sqrt{15.69^2 + 1.13^2} \angle \tan^{-1}\left(\frac{-1.13}{15.69}\right) \text{ A} = 15.73 \angle -4.12^\circ \text{ A}$$

$$d. i(t) = 460 \cos(500\pi t - 25^\circ) - 220 \sin(500\pi t + 15^\circ) \text{ A}$$

Note: can only add if same frequency

$$= 460 \angle -25^\circ - 220 \angle -75^\circ \text{ A}$$

$$= 416.90 - j194.4 - (56.94 - j212.5) \text{ A}$$

$$= 359.96 + j18.10 \text{ A}$$

$$= 360.4 \angle 2.88^\circ \text{ A}$$

4.38 Convert to polar

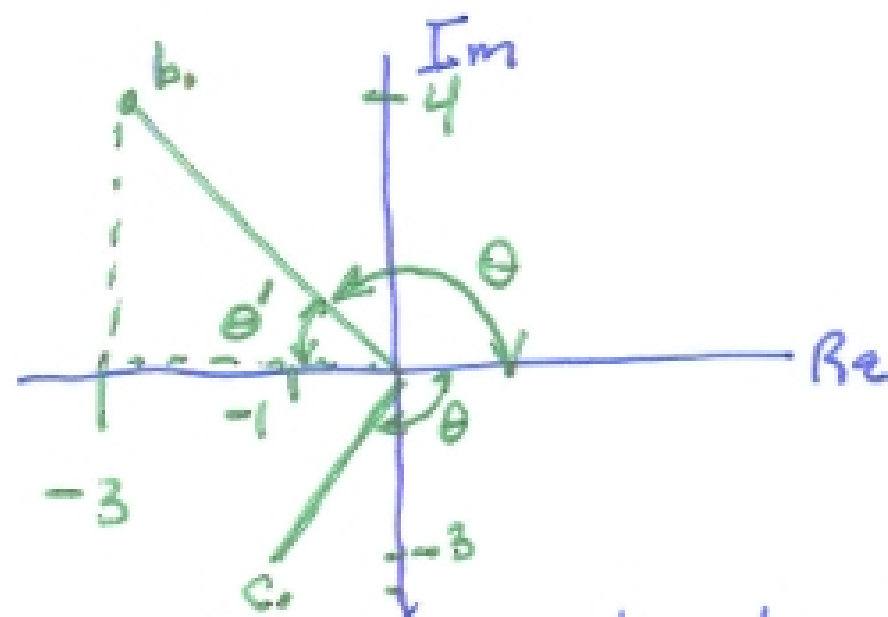
$$a. 4 + j4 = \sqrt{4^2 + 4^2} \angle \tan^{-1}\left(\frac{4}{4}\right) = 4\sqrt{2} \angle 45^\circ$$

$$b. -3 + j4 = \sqrt{3^2 + 4^2} \angle \tan^{-1}\left(\frac{4}{3}\right) = 5 \angle 126.9^\circ$$

$$= 5 \angle 2.21 \text{ rad}$$

$$c. j+2 - j4 - 3 = -1 - j3 = 3.16 \angle -108.4^\circ$$

Additional information for b & c



We are looking for θ .
Draw the number
in the complex plane
to better understand θ .

Solution ①: Find θ' in the triangle

$$\theta' = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

$$\theta = 180^\circ - 53.1^\circ = 126.9^\circ$$

Solution ②:

$$\tan(\theta'' + n\pi) = -\frac{4}{3} \text{ for } n = 0, \pm 1, \pm 2, \dots$$

Calculator returns $\theta'' = -53.1^\circ$ assuming $n = 0$
you now need to find the correct n so that
 $-180^\circ < \theta \leq 180^\circ$

$$n = 1 \rightarrow \theta = -53.1^\circ + 180^\circ = 126.9^\circ$$

(Same reasoning for c.)

4.39 Convert to polar, then compute product

$$a. (50 + j10)(4 + j8) = (50.99 \angle 11.3^\circ)(8.94 \angle 63.43^\circ) \\ = 456.1 \angle 74.7^\circ$$

Note that you can multiply rectangular numbers

$$(50 + j10)(4 + j8) = 50 \cdot 4 + j50 \cdot 8 + j10 \cdot 4 + j^2 10 \cdot 8 \\ = 200 + j400 + j40 - 80 \quad \leftarrow j^2 = -1 \\ = 120 + j440 = 456.1 \angle 1.3 \text{ rad} \\ = 456.1 \angle 74.75^\circ$$

$$b) (j2 - 2)(4 + j5)(2 + j7)$$

$$= (2.82 \angle 135^\circ)(6.40 \angle 51.34^\circ)(7.28 \angle 74.05^\circ) = 131.8 \angle 260.4^\circ$$

the angle is typically shown between $+180^\circ$ and -180°
 $260.4^\circ - 360^\circ = -99.6^\circ$

$$= 131.8 \angle -99.6^\circ$$

Using rectangular form

$$(j2 - 2)(4 + j5)(2 + j7) = (j2 - 2)(8 + j28 + j10 - 35) \\ = (j2 - 2)(-27 + j38) = -j54 - 76 + 54 - j76 \\ = -22 - j130 = 131.8 \angle 260.4^\circ = 131.8 \angle -99.6^\circ$$