

ECE 201 Spring 2010

Homework 37 Solutions

Problem 5

(a)

For Figure (a),

$$\begin{aligned} I_{eff}^2 &= \frac{1}{T} \int_0^T i^2(t) dt \\ &= \frac{1}{9} \left[\int_0^3 9 dt + \int_3^6 16 dt \right] \\ &= \frac{25}{3} \\ \Rightarrow I_{eff} &= 5/\sqrt{3} \end{aligned}$$

For Figure (b),

$$\begin{aligned} I_{eff}^2 &= \frac{1}{4} \left[\int_0^1 9 dt + \int_2^3 16 dt \right] \\ &= 25/4 \\ \Rightarrow I_{eff} &= 2.5 \end{aligned}$$

(b)

Current through R_L is given by $i(t) \times 60/(30 + 60) = (2/3)i(t)$. Power absorbed by R_L is then given by the square of the effective current through it times the resistance. Thus

$$\begin{aligned} P &= \frac{4}{9} \times \frac{25}{3} \times 30 \\ &= \frac{1000}{9} \text{ W} \end{aligned}$$

(c)

$$\begin{aligned} P &= \frac{4}{9} \times \frac{25}{4} \times 30 \\ &= \frac{250}{3} W \end{aligned}$$

Problem 7

In all parts of the problem, the following identities will be used,

$$\begin{aligned} \int_0^{2\pi/m} \cos(mnx) dx &= 0 \\ \int_0^{2\pi/m} \sin(mnx) dx &= 0 \end{aligned}$$

Here m and n are integers.

(a)

$$\begin{aligned} V_{1eff}^2 &= \frac{1}{T} \int_0^T v_1^2(t) dt \\ &= \frac{1}{T} \int_0^T [102 + 40 \cos(20t) + 2 \cos(40t)] dt \\ &= 102 \\ \Rightarrow V_{1eff} &= 10.0995 \end{aligned}$$

(b)

$$\begin{aligned} V_{2eff}^2 &= \frac{1}{T} \int_0^T v_2^2(t) dt \\ &= \frac{1}{T} \int_0^T [50\{1 + \cos(4t)\} + 12.5\{1 + \cos(8t)\} + 50\{\cos(6t) + \cos(2t)\}] dt \\ &= 62.5 \\ \Rightarrow V_{2eff} &= 7.9057 \end{aligned}$$

(c)

$$\begin{aligned} V_{3eff}^2 &= \frac{1}{T} \int_0^T v_3^2(t) dt \\ &= \frac{1}{T} \int_0^T [50\{1 + \cos(4t)\} + 36.4277\{1 + \cos(8t)\} + 6.2499\{1 - \cos(8t)\} + \dots] dt \\ &= 92.6776 \\ \Rightarrow V_{3eff} &= 9.6269 \end{aligned}$$

Problem 9

(a)

$$\begin{aligned}Z_L &= \left[\frac{1}{5} + j(30 \times 5 \times 10^{-3}) \right]^{-1} \\ &= 0.8(4 - j3) \\ \Rightarrow \mathbf{V}_L &= \mathbf{I}_{in} \times Z_L \\ &= \frac{4}{\sqrt{2}}(4 - j3) \\ \Rightarrow v_L(t) &= 20 \cos(30t - 36.87^\circ) \text{ V}\end{aligned}$$

(b)

$$\begin{aligned}P_{inst}(t) &= i_{in}(t)v_L(t) \\ &= 100 \cos(30t) \cos(30t - 36.87^\circ) \text{ W} \\ P_{average} &= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \\ &= 50 \cos 36.87^\circ \\ &= 40 \text{ W}\end{aligned}$$

Problem 10

(a)

$$\begin{aligned}\mathbf{I}_s &= \frac{50 \angle -90^\circ}{6 + j12 - j4} \\ &= 5 \angle -143.13^\circ \text{ (magnitude = 5)}\end{aligned}$$

(b)

$$\begin{aligned}P_{average} &= V_{eff} I_{eff} \cos(\theta_v - \theta_i) \\ &= 5 \times 50 \cos(-53.13^\circ) \\ &= 150 \text{ W}\end{aligned}$$