

To Frame the World—25 Sept

- Kepler found orbit of Mars relative to earth's orbit.
- Goal was to measure the absolute distance (in miles or km) of the solar system
- Cassini & Richer 1672



Giovanni Domenico Cassini, (1625 - 1712)
engraving by N. Dupuis
www.slsu.edu/digitalcollections/hst/scientific-identity/fall/06/SIL14-C1-18a.jpg

Hipparchus measures the moon's distance ~200BC

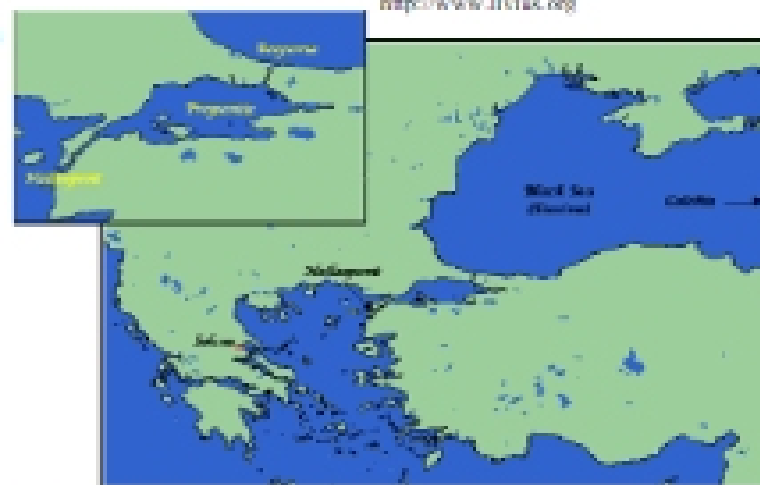
- At the Hellespont, the solar eclipse of 189BC was total. (Sparta defeated Athens there in 405 BC.)
- In Alexandria, the moon covered $\frac{1}{4}$ of the sun.
- Draw a picture to show the relationship between the sun, the moon, the two locations, and the difference between a total & $\frac{1}{4}$ eclipse. (The diameter of the sun is $\frac{1}{2}$ degree.)



<http://www.dixias.org>



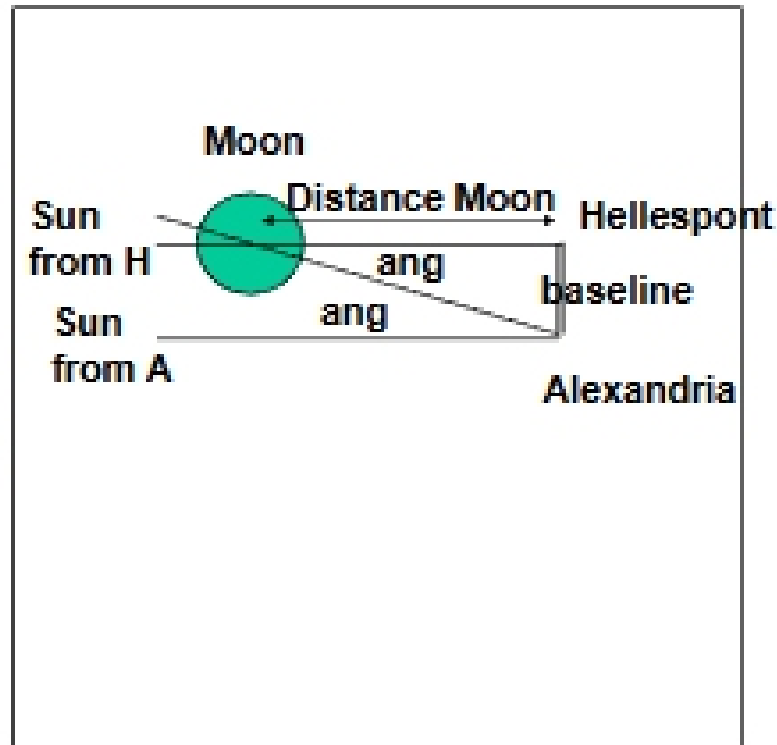
View in Alexandria.
Moon is offset by $\frac{1}{4}$
diameter of sun



<http://mkatz.wsh.washstate.edu/media/lecture/hellespont.gif>

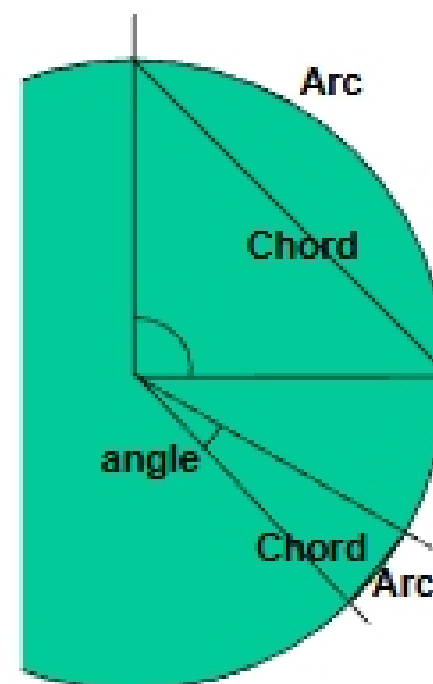
What triangle did Hipparchus use?

- Parts of triangle
 - Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
 - One leg of triangle is the baseline.
 - Other leg is distance to moon
- Method of parallax.
 - Angle is the "parallactic shift."



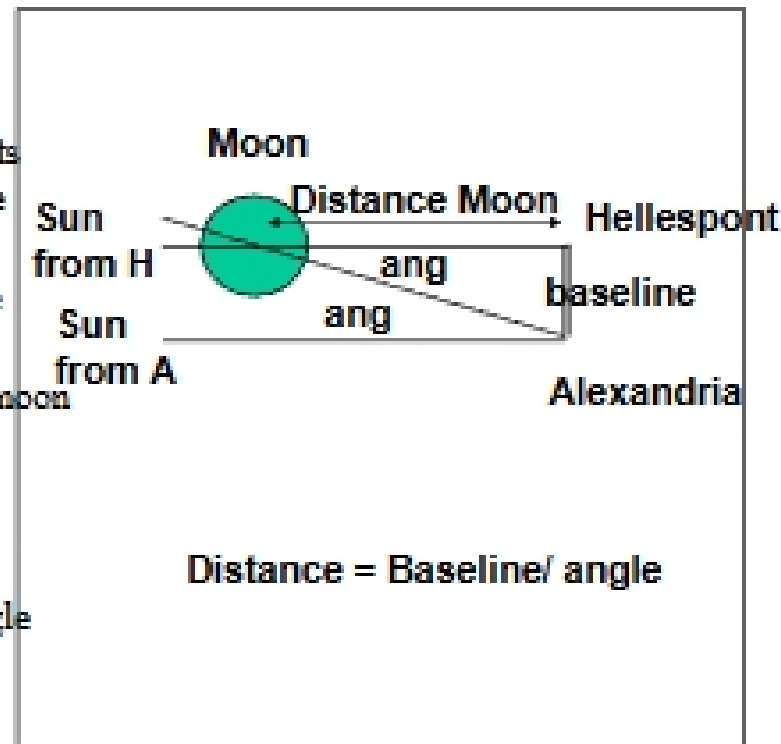
Small angle approximation

- Measure angles in radians
- Arc = radius \times angle
 - For entire circle,
 - Length of arc = circumference
 - Therefore angle = 2π for the circle
 - 2π radians = 360°
- For small angles, arc is approximately equal to the chord.
 - Chord = radius \times angle
- Application
 - Baseline = Distance \times angle



Method of parallax

- Parts of triangle
 - Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
 - One leg of triangle is the baseline.
 - Other leg is distance to moon
- Method of parallax.
 - Angle is the “parallactic shift”
 - Distance = Baseline/ angle



Difficulties

- Small angles are hard to measure
 - Naked eye $1/30^\circ = 1/1700 \text{ rad} = 6 \times 10^{-4} \text{ rad} = 600 \mu\text{rad}$
 - Modern telescope used under ideal conditions: $5 \mu\text{rad}$
 - Modern telescope with correction for atmospheric turbulence: $0.5 \mu\text{rad}$
 - Moon using Hellespont & Alexandria
 - Angle = baseline/distance = $1000\text{km} / 400,000\text{km}$
 - = $1/400 \text{ rad} = 2500 \mu\text{rad} (1/7^\circ)$
 - Mars
 - Angle = baseline/distance = $1000\text{km} / 80,000,000\text{km}$
 - = $13 \mu\text{rad}$ using Hellespont & Alexandria
- Need a reference nearby in the sky
 - Measuring with a reference on the ground is impossible.