251prob 10/27/04 (Open this document in 'Outline' view!)

H. Introduction to Probability

1. Experiments and Probability

Define a random experiment, a sample space, an outcome, a basic outcome

a. Definition and rules for Statistical Probability.

(i) If A_1 is impossible, $P(A_1) = 0$.

(ii) If A_1 is certain, $P(A_1) = 1$.

(iii) For any Outcome A_1 , $0 \le P(A_1) \le 1$.

(iv). If $A_1, A_2, A_3, \cdots\cdots, A_N$ represent all possible outcomes and are mutually exclusive, then $P(A_1) + P(A_2) + P(A_3) + \cdots\cdots P(A_N) = 1$.

b. An Event.

c. Symmetrical, Statistical and Subjective Probability.

2. The Venn Diagram.

A diagram representing events as sets of points or 'puddles.'

a. The Addition Rule. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(i) Meaning of Union (or).

$P(A \cup B)$ means the probability of A occurring or of B occurring or both. It always includes $P(A \cap B)$ if it exists.

(ii) Meaning of Intersection (and).

$P(A \cap B)$ means the probability of both A and B occurring. Note that if A and B are mutually exclusive $P(A \cap B) = 0$

Diagram for dice problems.

	1	2	3	4	5	6
1	•	•	•	•	•	•
2	•	•	•	•	•	•
3	•	•	•	•	•	•
4	•	•	•	•	•	•
5	•	•	•	•	•	•
6	•	•	•	•	•	•

b. Meaning of Complement. $P(\overline{A}) = 1 - P(A)$.

This event can be called $'not\ A'$. Note that if A and B are collectively exhaustive $P(A \cup B) = 1$. If A and B are both collectively exhaustive and mutually exclusive B is the complement of A.

c. Extended Addition Rule.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

3. Conditional and Joint Probability.

a. The Multiplication Rule $P(A \cap B) = P(A|B)P(B)$ or $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

The conditional probability of A given B , $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ is the probability of event A assuming that event B has occurred.

b. A Joint Probability Table.

What is the difference between joint, marginal and conditional probabilities? Remember that we cannot read a conditional probability directly from a joint probability table but must compute it using the second version of the Multiplication Rule.

c. Extended Multiplication Rule. $P(A \cap B \cap C) = P(C|A \cap B)P(B|A)P(A)$

d. Bayes' Rule. $P(B|A) = \dfrac{P(A|B)P(B)}{P(A)}$

4. Statistical Independence.

a. Definition: $P(A|B) = P(A)$

b. Consequence: $P(A \cap B) = P(A) \cdot P(B)$

c. Consequence: If A and B are independent so are $\overline{A}$ and $\overline{B}$, A and $\overline{B}$ etc.

5. Review.

Rule	In General	A and B mutual ly exclusive	A and B independent
Multiplication $P(A \cap B)$	$=P(A\|B)P(B)$ $=P(B\|A)P(A)$	$=0$	$=P(A)P(B)$
Addition $P(A \cup B)$	$=P(A)+P(B)-P(A \cap$	$=P(A)+P(B)$	$=P(A)+P(B)-P(A)$
Bayes' Rule $P(A\|B)$	$=\dfrac{P(B\|A)P(A)}{P(B)}$	$=0$	$=P(A)$
Bayes' Rule $P(B\|A)$	$=\dfrac{P(A\|B)P(B)}{P(A)}$	$=0$	$=P(B)$

I. Permutations and Combinations.

1. Counting Rule for Outcomes.

a. If an experiment has k steps and there are n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, etc. up to n_k possible outcomes on the k th step, then the total number of possible outcomes is the product $n_1 n_2 \cdots n_k$.

b. Consequence. If there are exactly n outcomes at each step, the total possible outcomes from k steps is $(n)^k$.

2. Permutations.

a. The number of ways that one can arrange n objects: $n!$

b. $P_r^n = \dfrac{n!}{(n-r)!}$ **Order counts!**

3. Combinations.

a. $C_r^n = \dfrac{n!}{(n-r)!r!}$ **Order doesn't count!**

b. Probability of getting a given combination

This is the number of ways of getting the specified combination divided by the total number of possible combinations. If there are a equally likely ways to get what you want and b equally likely possible outcomes, the probability of getting the outcomes you want is a/b. Example: If there is only one way to get 4 jacks from 4 jacks in a poker hand and C_1^{48} ways to get another card, $a = (1)C_1^{48}$ The number of ways to get a poker hand of 5 cards is $b = C_5^{52}$, so the probability of getting a poker hand with 4 jacks is $a/b = C_1^{48}/C_5^{52}$.