

Background Material

This handout summarizes all the background material I expect you to know when you start Math 19. You will be quizzed on this material on Monday, January 14. Many of the formulas in this handout are also available on the reference page at the very beginning of your textbook.

1 Simplifying Algebraic Expressions

1. You should know how to handle fractions (e.g., adding, multiplying, dividing, using common denominators, cancelling).
2. You should know how to factor a perfect square out of a square root. For example:

$$\begin{aligned}\sqrt{18} &= 3\sqrt{2}, \\ \sqrt{x^3 - x^2} &= x\sqrt{1 - x}.\end{aligned}$$

3. Throughout the course, we will have equations that have **constants** in them. For example, I might say: let $y = cx$, where c is a constant. This means that c is some fixed number, but we do not know what it is. In this case, we know that the graph of $y = cx$ is a straight line going through the point $(0, 0)$ with slope c (we just don't know what that slope c is).

We often state rules using constants. For example, the Quadratic Formula says that if $0 = ax^2 + bx + c$, where a , b , and c are constants, and $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For example, the Quadratic Formula is still true if a , b , and c are replaced by any numbers (assuming $a \neq 0$).

However, the Quadratic Formula is *not* true if a is replaced by x , because x is not a constant. That is to say, if $0 = x \cdot x^2 + bx + c$, the Quadratic Formula is not the correct way to solve for x .

2 Solving Equations and Inequalities

1. Avoid dividing by zero. For example, to solve

$$x^3 - x = x,$$

you cannot divide everything by x and say that

$$x^2 - 1 = 1,$$

and so $x = \pm\sqrt{2}$. The reason you cannot do this is that x might equal 0, and in fact, $x = 0$ is a solution to the original equation. So instead of dividing by x , you collect everything to one side of the equation and then factor out an x :

$$\begin{aligned}x^3 - x &= x \\x^3 - 2x &= 0 \\x(x^2 - 2) &= 0 \\x(x + \sqrt{2})(x - \sqrt{2}) &= 0\end{aligned}$$

So we see $x = 0$ or $x = \pm\sqrt{2}$.

2. You should know how to solve equalities and inequalities with absolute values. For example:

$$\begin{aligned}|x - 3| &= 4 \\x - 3 &= 4 \quad \text{or} \quad x - 3 = -4 \\x &= 7 \quad \text{or} \quad x = -1\end{aligned}$$

$$\begin{aligned}|x - 3| &\leq 4 \\-4 &\leq x - 3 \leq 4 \\-1 &\leq x \leq 7\end{aligned}$$

$$\begin{aligned}|x - 3| &\geq 4 \\x - 3 &\leq -4 \quad \text{or} \quad x - 3 \geq 4 \\x &\leq -1 \quad \text{or} \quad x \geq 7\end{aligned}$$

3. When solving equalities and inequalities, you have to be careful with things like square roots. For example:

$$\begin{aligned}x^2 &= 4 \text{ has two solutions, } x = \pm 2, \\x^2 &\leq 4 \text{ has the solution } -2 \leq x \leq 2, \text{ not } x \leq 2, \\x^2 &\geq 4 \text{ has the solution } x \leq -2 \text{ or } x \geq 2, \text{ not } x \geq 2.\end{aligned}$$

4. Remember that when solving inequalities, if you divide by a negative number, you need to flip the inequality sign around:

$$\begin{aligned}-2x &\leq 4 \\x &\geq -2\end{aligned}$$

3 Functions

3.1 Polynomials

1. A **linear polynomial** has the form $y = ax + b$ and is the equation of a straight line. A **quadratic polynomial** has the form $y = ax^2 + bx + c$ and is the equation of a parabola. A **cubic polynomial** has the form $y = ax^3 + bx^2 + cx + d$ and is the equation of a cubic curve.
2. Given two points in the plane, you should feel comfortable finding the slope of the line passing through these two points. If the points are (x_1, y_1) and (x_2, y_2) , then the slope is defined as

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}.$$

It is easy to remember this formula as “rise over run.”

3. The equation of a straight line with slope m and y -intercept b is $y = mx + b$. The equation of a straight line that goes through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

4. The **y -intercept(s)** of a curve is/are the y -value(s) at which the curve intersects the y -axis. This can be found by letting $x = 0$ and finding the value(s) of y . The **x -intercept(s)** of a curve is/are the x -value(s) at which the curve intersects the x -axis. This can be found by letting $y = 0$ and finding the value(s) of x .

3.2 Trigonometric Functions

1. In this class (and in future math classes) you should always use radians, never degrees.
2. Given the right triangle below, we have the formulas

$$\begin{aligned}\sin a &= \frac{y}{z} \\ \cos a &= \frac{x}{z} \\ \tan a &= \frac{y}{x}\end{aligned}$$

