Econ 311: Limited Dependent Variable Models Background

A key result is

$$f(y|y\ >\ a) = \frac{f(y)}{\Pr\left[y > a\right]} = \frac{\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-1}{2\sigma^2}\left[y-\mu\right]^2\right)}{1-\Phi\left(\frac{y-a}{\sigma}\right)}$$

$$= \frac{\frac{1}{\sigma}\left[\frac{1}{\sqrt{2\pi}}\exp\left(\frac{1}{2}\left[\frac{y-\mu}{\sigma}\right]^2\right)\right]}{1-\Phi\left(\frac{y-a}{\sigma}\right)} = \frac{\frac{1}{\sigma}\phi(\frac{y-\mu}{\sigma})}{1-\Phi(\alpha)}$$

where now and henceforth $\alpha = (a-\mu)/\sigma$.

Sampling schemes.

Random sample.

We draw observations on y randomly

from the population of y. For all observations we see the exact value of y. If y is iid with density $f(y)$, then the joint denisty of the sample $\{y_i\}$ is $\Pi_i f(y_i)$.

Truncated sample.

We observe the exact value of y iff $y > a$. We have no observations for $y \leq a$. We don't know the number of observations for which $y \leq a$. If $y \sim NID(\mu, \sigma)$, then the joint density of the sample $\{y_i\}$ is

$$\prod_{i=1}^{n} \frac{\frac{1}{\sigma}\phi\left(\frac{y_i - \mu}{\sigma}\right)}{[1 - \Phi(\alpha)]}$$

Censored sample.

We have a sample of size n, of which n_1 are such that $y \leq a$ and $n_2 = n - n_1$ are such that $y > a$, and only for these n_2 observations are the exact values of y known. For the n_1 observations for which $y \leq a$, we observe (or set) $y = a$. If $y \sim NID(\mu, \sigma)$, then the joint density of the sample $\{y_i\}$ is

$$[\Phi(\alpha)]^{n_1} \prod_{i=1}^{n_2} \phi(y_i).$$

Truncated model.