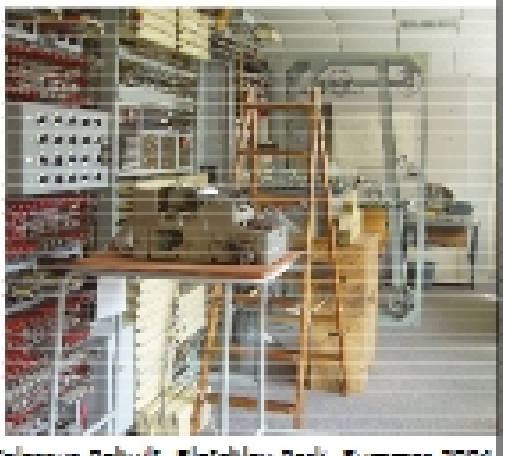
Lecture 17: Double Deltas and Banburismus



Colossus Rebuilt, Bletchley Park, Summer 2004

We'll finish up the tree sorting next week

Today's notes: on-line only (links)

CS150: Computer Science
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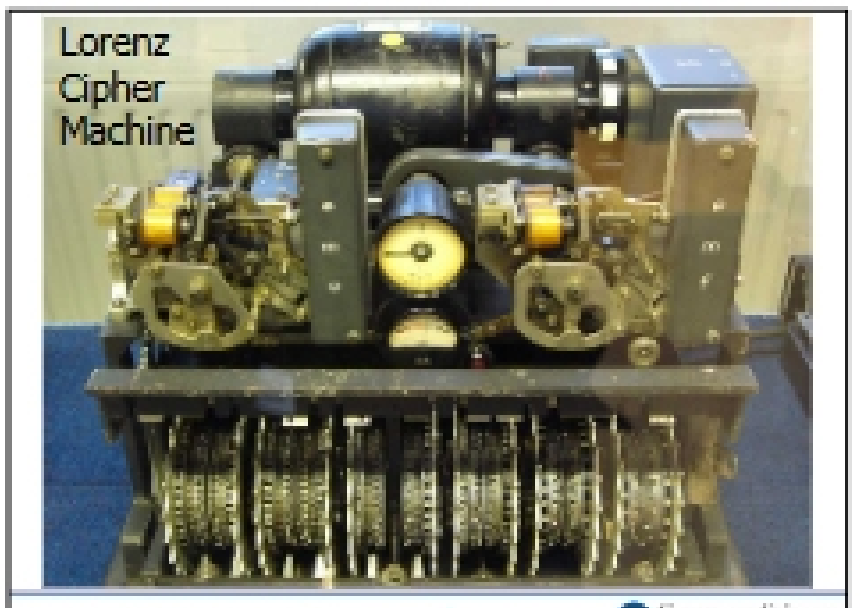


Lorenz Wheels

12 wheels
501 pins total (set to control wheels)

Work to break in $\Theta(p^n)$ so real Lorenz is $41^{12}/5^3$ – 1 quintillion (10^{18}) times harder!

Breaking Fish

- GCHQ learned about first Fish link (Tunny) in May 1941
  - Intercepted unencrypted Baudot-encoded test messages
- August 30, 1941: Big Break!
  - Operator retransmits failed message with same starting configuration
  - Gets lazy and uses some abbreviations, makes some mistakes
    - SPRUCHNUMMER/SPRUCHNR. (Serial Number)

"Two Time" Pad

- Allies have intercepted:

  $C1 = M1 \oplus \mathbf{K1}$

  $C2 = M2 \oplus \mathbf{K1}$

  Same key used for both (same starting configuration)
- Breaking message:

  $C1 \oplus C2 = (M1 \oplus \mathbf{K1}) \oplus (M2 \oplus \mathbf{K1})$

  $= (M1 \oplus M2) \oplus (\mathbf{K1} \oplus \mathbf{K1})$

  $= M1 \oplus M2$

"Cribs"

- Know: C1, C2 (intercepted ciphertext)

  $C1 \oplus C2 = M1 \oplus M2$
- Don't know M1 or M2
  - But, can make some guesses (cribs)
    - SPRUCHNUMMER
    - Sometimes allies moved ships, sent out bombers to help the cryptographers get good cribs
- Given guess for M1, calculate M2

  $M2 = C1 \oplus C2 \oplus M1$
- Once guesses that work for M1 and M2

  $K1 = M1 \oplus C1 = M2 \oplus C2$

Reverse Engineering Lorenz

- From the 2 intercepted messages, Col. John Tiltman worked on guessing cribs to find M1 and M2: 4000 letter messages, found 4000 letter key K1
- Bill Tutte (recent Chemistry graduate) given task of determining machine structure
  - Already knew it was 2 sets of 5 wheels and 2 wheels of unknown function
  - Six months later new machine structure likely to generate K1

Intercepting Traffic

- Set up listening post to intercept traffic from 12 Lorenz (Fish) links
  - Different links between conquered capitals
  - Slightly different coding procedures, and different configurations
- 600 people worked on intercepting traffic

Breaking Traffic

- Knew machine structure, but a different initial configuration was used for each message
- Need to determine wheel setting:
  - Initial position of each of the 12 wheels
  - 1271 possible starting positions
  - Needed to try them fast enough to decrypt message while it was still strategically valuable

This is what you did for PS4 (except with fewer wheels)

Recognizing a Good Guess

- Intercepted Message (divided into 5 channels for each Baudot code bit)

  $Z_c = z_0 z_1 z_2 z_3 z_4 z_5 z_6 z_7 \ldots$

  $z_{c,i} = m_{c,i} \oplus x_{c,i} \oplus s_{c,i}$

  Message Key (parts from S-wheels and rest)
- Look for statistical properties
  - How many of the $z_{c,i}$'s are 0? ½ (not useful)
  - How many of $(z_{c,i+1} \oplus z_{c,i})$ are 0? ½

Double Delta

$$\Delta Z_{c,i} = Z_{c,i} \oplus Z_{c,i+1}$$

Combine two channels:

$$\Delta Z_{1,i} \oplus \Delta Z_{2,i} = \Delta M_{1,i} \oplus \Delta M_{2,i} > ½ \text{ Yippee!}$$
$$\oplus \Delta X_{1,i} \oplus \Delta X_{2,i} = ½ \text{ (key)}$$
$$\oplus \Delta S_{1,i} \oplus \Delta S_{2,i} > ½ \text{ Yippee!}$$

Why is $\Delta M_{1,i} \oplus \Delta M_{2,i} > ½$

Message is in German, more likely following letter is a repetition than random

Why is $\Delta S_{1,i} \oplus \Delta S_{2,i} > ½$

S-wheels only turn when M-wheel is 1

Actual Advantage

- Probability of repeating letters

  $\text{Prob}[\Delta M_{1,i} \oplus \Delta M_{2,i} = 0] \sim 0.614$

  3.3% of German digraphs are repeating
- Probability of repeating S-keys

  $\text{Prob}[\Delta S_{1,i} \oplus \Delta S_{2,i} = 0] \sim 0.73$

$\text{Prob}[\Delta Z_{1,i} \oplus \Delta Z_{2,i} \oplus \Delta X_{1,i} \oplus \Delta X_{2,i} = 0]$

$= 0.614 * 0.73 + (1-0.614) * (1-0.73)$

ΔM and S are 0 ΔM and S are 1

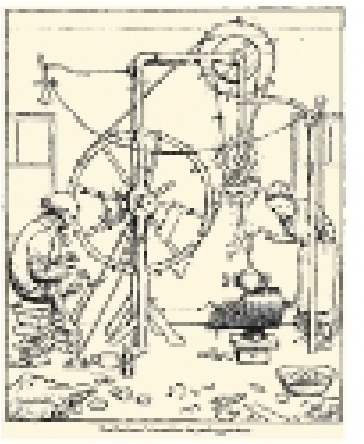
= 0.55 if the wheel settings guess is correct (0.5 otherwise)

Using the Advantage

- If the guess of **X** is correct, should see higher than ½ of the double deltas are 0
- Try guessing different configurations to find highest number of 0 double deltas
- Problem:
  - # of double delta operations to try one config
  - = length of Z * length of X
  - = for 10,000 letter message = 12 M for each setting * 7 ⊕ per double delta
  - = 89 M ⊕ operations

Heath Robinson

- Dec 1942: Decide to build a machine to do these ⊕s quickly, due June 1943
- Apr 1943: first Heath Robinson machine is delivered!
- Intercepted ciphertext on tape:
  - 2000 characters per second (12 miles per hour)
  - Needed to perform 7 ⊕ operations each ½ ms

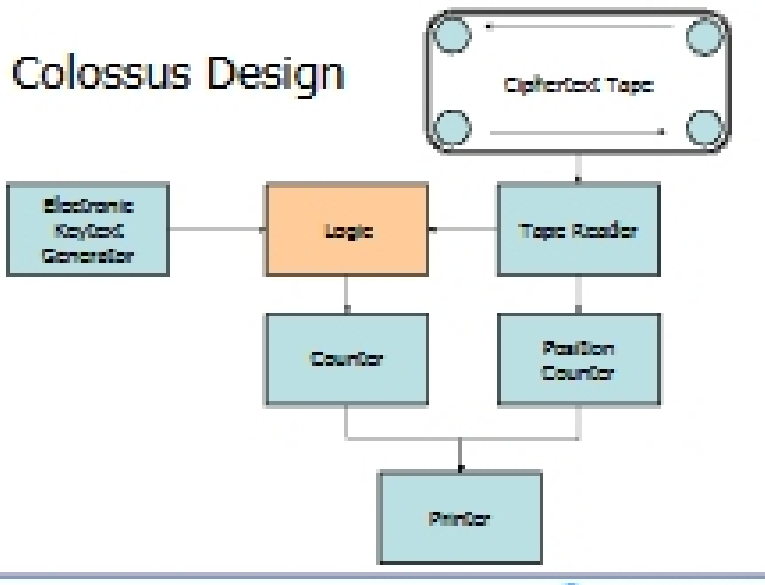


Heath Robinson, British Cartoonist (1872-1944)

Colossus

- Heath Robinson machines were too slow
- Colossus designed and first built in Jan 1944
- Replaced keytext tape loop with electronic keytext generator
- Speed up ciphertext tape:
  - 5,000 chars per second = 30 mph
  - Perform 5 double deltas simultaneously
  - Speedup = 2.5X for faster tape * 5X for parallelism

Colossus Design



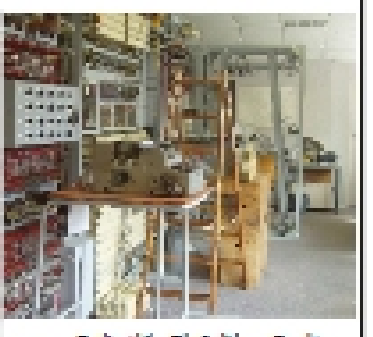
Impact on WWII

- 10 Colossus machines operated at Bletchley park
  - Various improvements in speed
- Decoded 63 million letters in Nazi command messages
- Learned German troop locations to plan D-Day (knew the deception was working)

Colossus History

Kept secret after the war, all machines destroyed

During WWII



Rebuild, Bletchley Park, Summer 2004