Take that, Bell's Inequality!

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Abstract

Bell's inequality was tested using the CHSH method. Entangled photons were produced from two different laser beams by passing light from Type I BBO crystals and detected using avalanche photodiodes. Values for S as high as 2.30 were obtained.

Background

One of the major differences between quantum mechanics and previous physical theories is its probabilistic nature. If the state of a classical system is completely known at some time then, in principle, it is possible to develop equations of motion for that system that will determine the values for any observable quantity at any time in the future. However, the most that quantum mechanical equations of motion can provide is the probability that a given observable will take a particular value.

This led some of the pioneers of the field to suspect that quantum mechanics was an incomplete theory, and that there were other, "hidden", variables that would allow one to completely determine the final state of a quantum system in the same way as a classical one.

In support of this view, Einstein, Podolsky, and Rosenberg proposed a thought experiment in 1936 using the phenomenon of entanglement. Quantum mechanics allows two particles that interact with each other to be put into a state in which the two particles cannot be treated as separate entities. Measurements of a property of one of the particles will still be probabilistic according to quantum mechanics, but they will be absolutely correlated with properties of the other particle. This condition persists even if the particles are subsequently separated from each other.

The argument is this, then: assuming the laws of physics are locally deterministic, that is, that the behavior of an isolated system will only depend on the state of the system, then any correlation between two isolated systems must be the result of some other information stored in the two systems. In other words, since two entangled particles separated by a large distance cannot communicate with each other, there must be some hidden variables responsible for the correlations seen when the particles are measured. Since the wave functions of quantum mechanics do not provide this information, quanum mechanics must be an incomplete theory.

In 1964, John Stewart Bell published a method to determine the validity of this argument. He showed that, in any physical system governed by local variables, the correlations observed between two systems must satisfy certain inequalities, regardless of the actual physics involved. However, under certain circumstances quantum mechanics predicts results that violate those inequalities. This means that, if quantum mechanics is correct, then there cannot be any additional theory based on hidden variables that will make it complete, and further that it is possible to determine experimentally whether this is the case!

Procedure

A test of Bell's inequality requires the production and observation of two particles in an entangled state. We create entangled photons by taking advantage of Spontaneous Parametric Down Conversion (SPDC), a nonlinear optical process in which a single high-frequency photon is annihilated and two lower-frequency photons are created. We use a Type I beta barium borate (BBO) crystal, which produces will convert an input photon into two photons (traditionally called Signal and Idler) of the same (unknown) polarization, a process illustrated in figure 2. The resulting entangled pair is therefore in the state

$$|\Phi\rangle = \frac{1}{2}\left(|V_S\rangle|V_I\rangle + |H_S\rangle|H_I\rangle\right) \tag{1}$$

In the first of our two configurations, showin in figure 3, light is produced by a $100mW$, $363.8nm$ Argon-ion laser. This light passes through a birefringent quartz waveplate, which serves to ensure that equal amounts of horizontally and vertically polarized light reach the BBO crystal. The main beam passes through the BBO and into a beam stop. Avalanche photodiodes (APDs) are positioned equidistant on either side of the beam; this ensures, due to conservation of momentum, that they will capture pairs of photons with nearly the same wavelength; a $10nm$ band-pass filter restricts this further. Variable-angle polarizers are also placed between the crystal and the two APDs. The APDs are then connected to a coincidence counter, which will track the number of occasions when photons arrive at the detectors simultaneously. The second configuration is identical to the first, with the following exceptions: the laser used is a $10mw$ diode laser, and the $10nm$ filters are omitted.

Every measurement in this setup will be taken for specific values of the angles α and β of the polarizers A and B. Because the coincidence counter has a finite resolution Δt ($26ns$ for the argon laser setup, and $40ns$ for the diode laser), there will be a small number of accidental coincidences observed when two non-entangled photons arrive at the detectors close enough together that they appear simultaneous, which may be estimated by

$$N_{acc} = N_A N_B \Delta t, \tag{2}$$

where N_A and N_B are the count rates at detectors A and B, respectively. We take three measurements of the coincidence count rate N for each setting, and the net count rate $N(\alpha, \beta)$ is then given by

$$N(\alpha, \beta) = \langle N \rangle - N_A N_B \Delta t \tag{3}$$

Bell's original inequality only holds for pairs of two-state systems, e.g. spin-$\frac{1}{2}$ particles. Therefore, we use the related result called the CHSH inequality, after Clauser, Horne, Shimony, and Holt. This method requires measurements of the coincidence count rate for 16

	a	b
θ	$-45°$	$-22.5°$
$\theta_\perp$	$0°$	$22.5°$
θ'	$90°$	$67.5°$
$\theta'_\perp$	$135°$	$112.5°$

Figure 1: Angles for CHSH bell measurements

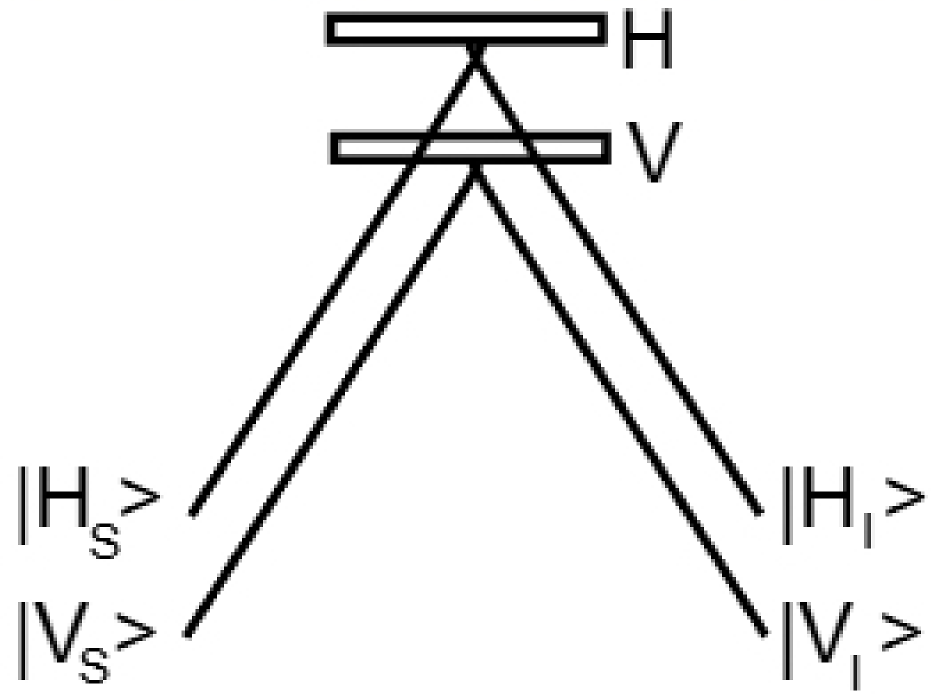

Figure 2: Entangled Photon Production in Type I BBO crystal

polarizer settings, chosen to maximize the deviation of the quantum prediction from the classical.

For a given angle pair (α, β) and the associated $(\alpha_\perp, \beta_b ot)$, we define

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha,\beta_\perp) - N(\alpha_\perp,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta)} \tag{4}$$

Then,

$$S = |E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \tag{5}$$

If $S > 2$, then Bell's inequality has been violated, and the behavior of the system cannot be explained by any locally deterministic model. For the angles tabulated in figure 1, quantum mechanics predicts a value of $S = 2\sqrt{2}$.

Testing for violations of Bell's inequalities is a three-step process: First, we attempt to align the wave plate so that it will produce equal amounts of horizontally and vertically polarized photons. Second, we rotate the polarizers through a full circle, and track the coincidence count rate as a function of relative polarizer angle; we expect a cosine-squared dependence. Third, we make the series of measurements at 16 angle settings specified by the CHSH method, and use them to construct Bell's Inequality.