

CS 2710 Foundations of AI
Lecture 18

**Inference in Bayesian belief
networks**

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Tic-Tac-Toe competition

- **Three winners:**

Tic-Tac-Toe competition

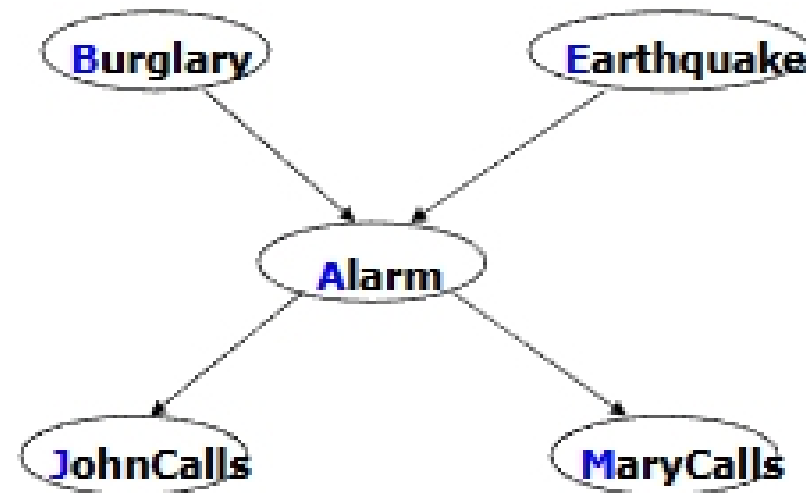
- **Three winners:**
 - Bryan Mills
 - Yaw Gyamfi
 - Lei Jin

Inference in Bayesian networks

- BBN models compactly the full joint distribution by taking advantage of existing independences between variables
- Simplifies the acquisition of a probabilistic model
- But we are interested in solving various **inference tasks**:
 - **Diagnostic task. (from effect to cause)**
$$\mathbf{P}(\textit{Burglary} \mid \textit{JohnCalls} = T)$$
 - **Prediction task. (from cause to effect)**
$$\mathbf{P}(\textit{JohnCalls} \mid \textit{Burglary} = T)$$
 - **Other probabilistic queries** (queries on joint distributions).
$$\mathbf{P}(\textit{Alarm})$$
- **Main issue:** Can we take advantage of independences to construct special algorithms and speeding up the inference?

Inference in Bayesian network

- **Bad news:**
 - Exact inference problem in BBNs is NP-hard (Cooper)
 - Approximate inference is NP-hard (Dagum, Luby)
- **But** very often we can achieve significant improvements
- Assume our Alarm network



- Assume we want to compute: $P(J = T)$

Inference in Bayesian networks

Computing: $P(J = T)$

Approach 1. Blind approach.

- Sum out all un-instantiated variables from the full joint,
- express the joint distribution as a product of conditionals

$$\begin{aligned}
 P(J = T) &= \\
 &= \sum_{b \in \{T, F\}} \sum_{e \in \{T, F\}} \sum_{a \in \{T, F\}} \sum_{m \in \{T, F\}} P(B = b, E = e, A = a, J = T, M = m) \\
 &= \sum_{b \in \{T, F\}} \sum_{e \in \{T, F\}} \sum_{a \in \{T, F\}} \sum_{m \in \{T, F\}} P(J = T | A = a) P(M = m | A = a) P(A = a | B = b, E = e) P(B = b) P(E = e)
 \end{aligned}$$

Computational cost:

Number of additions: ?

Number of products: ?