Fitting Consumer Panel Data using Bayesian Multinomial Probit Model

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Abstract

Customer choices of which goods to buy is one of the most interesting topics to marketing researchers because there are at least two alternatives, in general, in the choice sets. In particular, which brand is preferred and to what extent the brand is preferred to another brand have been fundamental problems to the marketing researchers and practitioners. In this project, we used data that has more than three alternative brands in the choice set. And the choice is a binary discrete variable since it the case is to make a purchase or not. Thus we considered multinomial probit model for this research. We fit the Bayesian multinomial probit model using Markov Chain Monte Carlo for the 'margarine' data in R-package bayesm.

1. Introduction

Customer's brand choice describes each person's or household's choice among alternatives, competing brands. Consider, for example, one who wants to buy a 6-pack beer is now standing in front of beer section at Hy-Vee Liquor. There are Samuel Adams Winter Ale, Heineken, Blue Moon, Fat Tire, Hoegaarden, etc. Unfortunately, among several options he can buy only one pack of those candidates. This is the context where customer makes their brand choice in everyday life, and at the same time, the procedure that most marketing practitioners and researchers want to investigate.

The choice set, in general, contains following characteristics. (Train 2003) The alternatives should be mutually exclusive from the decision maker's perspective. That is, selecting one option eliminates the possibilities selecting others. In addition, the number of alternatives is finite. As we can notice from the previous beer example, customer brand choice is discrete – either buying or not -and mostly having more than three alternatives in the choice set.

The logit model used to be the universal model to explain discrete choice models in marketing. However, it is limited because it cannot represent random taste variation and it cannot be used with panel data when unobserved factors are correlated over time for each decision maker. (Train 2003) He addresses that the probit model enables us to handle these drawbacks. Therefore, given choices between logit and probit for this project, we decided to use Bayesian multinomial probit model. Also, in order to fit Bayesian multinomial probit model via Markov Chain Monte Carlo, we used R package named **MNP.**

2. Methods

MNP is used to fit Bayesian multinomial probit model via Markov chain Monte Carlo. The efficient marginal data augmentation algorithm developed by Imain and van Dyk (2005) is used in the computation.

2.1 Model Specification: Multinomial probit model

In marketing literature, the data named "Scanner data" is often studied in an attempt to predict what consumer's next purchase using modeling techniques. Scanner data mostly have n observation with $p > 2$ choices and k covariates. Because outcomes of choices are discrete and we have more than two alternatives, the multinomial probit model is widely used to fit the data in marketing.

Under the multinomial probit model, we assume a multivariate normal distribution on the latent variables, $W_i = (W_{i1}, ..., W_{i,p-1})$.

$$W_i = X_i\beta + e_i,\ e_i \sim N(0, \Sigma), \text{ for } i = 1, \dots, n.$$

where X_i is a $(p-1)\times k$ matrix of covariates, β is $k\times 1$ vector of fixed coefficients, e_i is $(p-1)\times 1$ vector of disturbances, and Σ is $(p-1)\times(p-1)$ positive definite matrix. To identify the model, the first diagonal element of the covariance matrix should be equal to 1. The index of choice of individual *i* among the alternatives in the choice set is the response variable in the model, denoted Y_i. Y_i is the function of W_i, the latent variable defined above. The response variable Y_i is defined as following:

$$\begin{aligned} Y_i(W_i) &= 0 \ \textit{if} \ \max(W_i) < 0 \\ &= j \ \textit{if} \ \max(W_i) = W_{ij} > 0 \end{aligned} \quad \text{for i } = 1,\dots,\text{n and j}=1,\dots,\text{p-1}$$

where Y_i equal to 0 indicates a base category.

2.2 Model Specification: Multinomial probit model with ordered preference

Intuitively, it can be understood that individuals would choose an alternative *j* over an alternative *j'* if they have more preference on the alternative *j* than *j'*. In microeconomic theory, two axioms of preferences, named comparability and transitivity must hold in the study of choice models.

To explain the preference of individuals more formally, let's denote the outcome of choices as $\Pr(Y^* = j \mid X^*, Y) = \int \Pr(Y^* = j \mid X^*, \beta, \Sigma, Y)\, p(\beta, \Sigma \mid Y)\, d(\beta, \Sigma)$ where *i=1,…,n* represents individuals and *j=1,…,p-1* stands for choice alternatives. If $Yi_j > Y_{ij'}$ for some $j \neq j'$, you can say that the alternative j is preferred to j' for individual *i*. If $Yi_j = Y_{ij'}$ for some $j \neq j'$, the alternatives *j* and *j'* are different to individual *i*. To impose the comparability to the preference $Yi_j \leq Yi_{j'}$ and $Yi_j \geq Y_{ij'}$ must hold. Also, for preference being transitive: for any *j*, *j'*, and *j''*