Chapter 7 Boundary Layer Theory

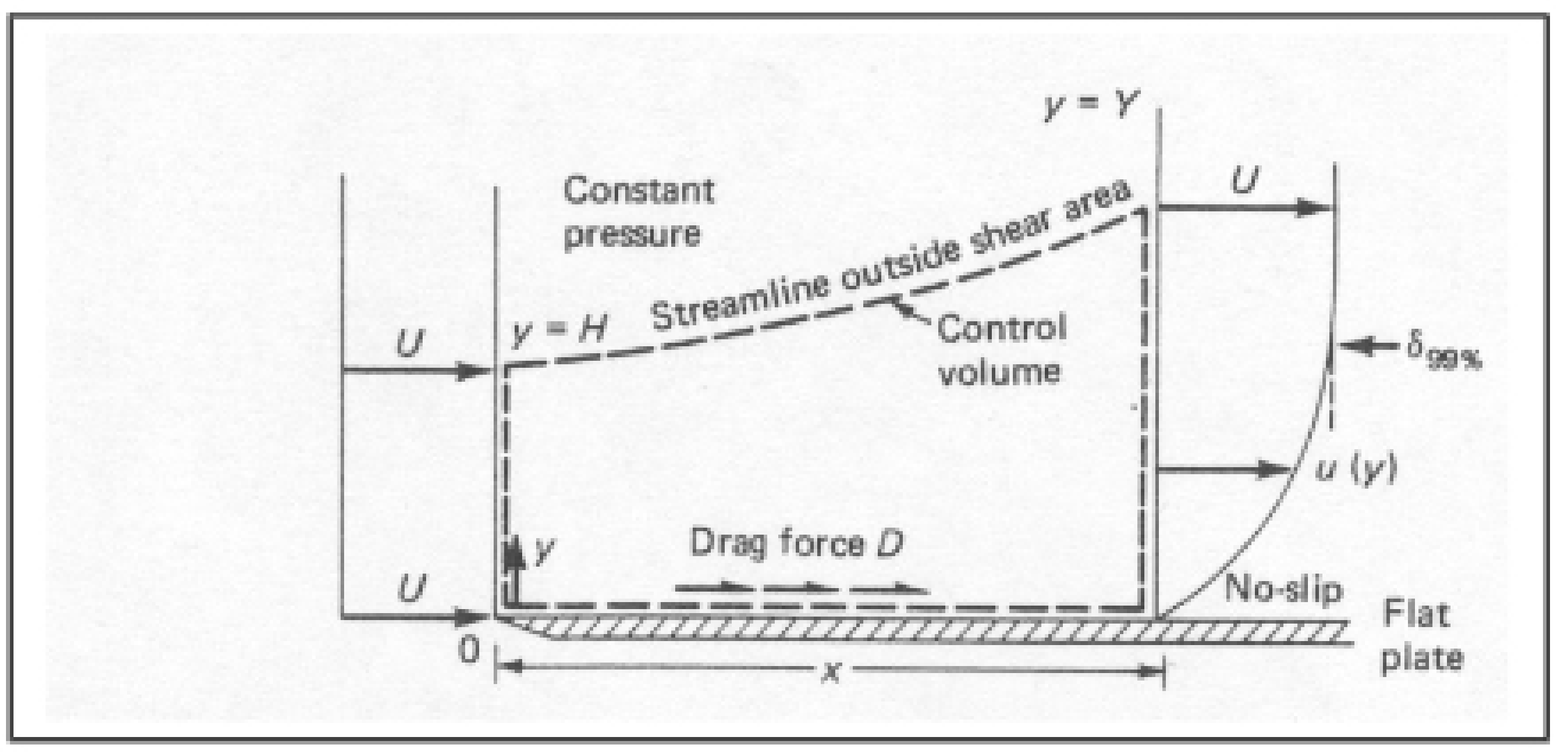
7.1. Introduction:

Boundary layer flows: External flows around streamlined bodies at high Re have viscous (shear and no-slip) effects confined close to the body surfaces and its wake, but are nearly inviscid far from the body.

Applications of BL theory: *aerodynamics* (airplanes, rockets, projectiles), *hydrodynamics* (ships, submarines, torpedoes), *transportation* (automobiles, trucks, cycles), *wind engineering* (buildings, bridges, water towers), and *ocean engineering* (buoys, breakwaters, cables).

7.2 Flat-Plate Integral Analysis & Laminar approximate solution

To gain much insight and quantitative information about boundary layers by making a broad-brush momentum analysis of the flow of a viscous fluid at high Re past a flat plate.



Boundary-layer thickness arbitrarily defined by y = $\delta_{99\%}$ (where, $\delta_{99\%}$ is the value of y at u = 0.99U). Streamlines outside $\delta_{99\%}$ will deflect an amount δ^* (**the displacement thickness**). Thus the streamlines move outward from $y = H$ at $x = 0$ to $y = Y = \delta = H + \delta^*$ at $x = x_1$.

<u>Conservation of mass</u>:

$$\int_{CS} \rho \mathbf{V} \cdot d\mathbf{A} = 0 = \int_0^{H+\delta^*} \rho u dy - \int_0^H \rho U dy$$

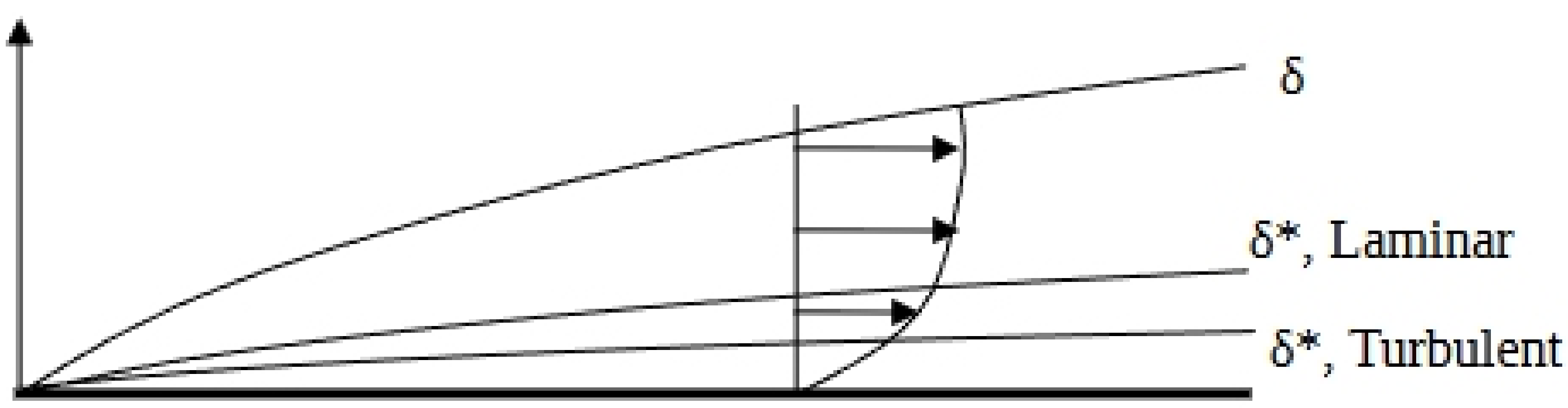
Assuming incompressible flow (constant density), this relation simplifies to

$$UH = \int_0^Y u dy = \int_0^Y (U + u - U) dy = UY + \int_0^Y (u - U) dy$$

Note: $Y = H + \delta^*$, we get the definition of displacement thickness:

$$\delta^* = \int_0^Y \left(1 - \frac{u}{U}\right) dy$$

δ^* is an important measure of effect of BL on external flow. To see this more clearly, consider an alternate derivation based on an equivalent discharge argument:



$$\underbrace{\int_{\delta^*}^{\delta} U dy}_{\text{Inviscid flow about } \delta^* \text{ body}} = \int_0^{\delta} u dy$$

(a) Discharge between δ^* and δ of inviscid flow=actual discharge, inviscid flow rate about displacement body = equivalent viscous flow rate about actual body

$$\int_0^{\delta} U dy - \int_0^{\delta^*} U dy = \int_0^{\delta} u dy \Rightarrow \delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

w/o BL - displacement effect=actual discharge

For 3D flow, in addition to (a), it must also be explicitly required that δ^* is a stream surface of the inviscid flow continued from outside of the BL.

<u>Conservation of x-momentum</u>:

$$\sum F_x = -D = \int_{CS} u \rho \underline{V} . d\underline{A} = -\int_0^H U(\rho U dy) + \int_0^Y u(\rho u dy)$$

$Drag = D = \rho U^2 H - \int_0^Y \rho u^2 dy$ = Force on plate = - Force on CV

Again assuming constant density and introducing: $H = \int_0^Y \frac{u}{U} dy$

$$D = \rho U^2 \int_0^Y u/U dy - \int_0^Y u^2 dy = \int_0^x \tau_w dx$$

$$\frac{D}{\rho U^2} = \theta = \int_0^Y \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

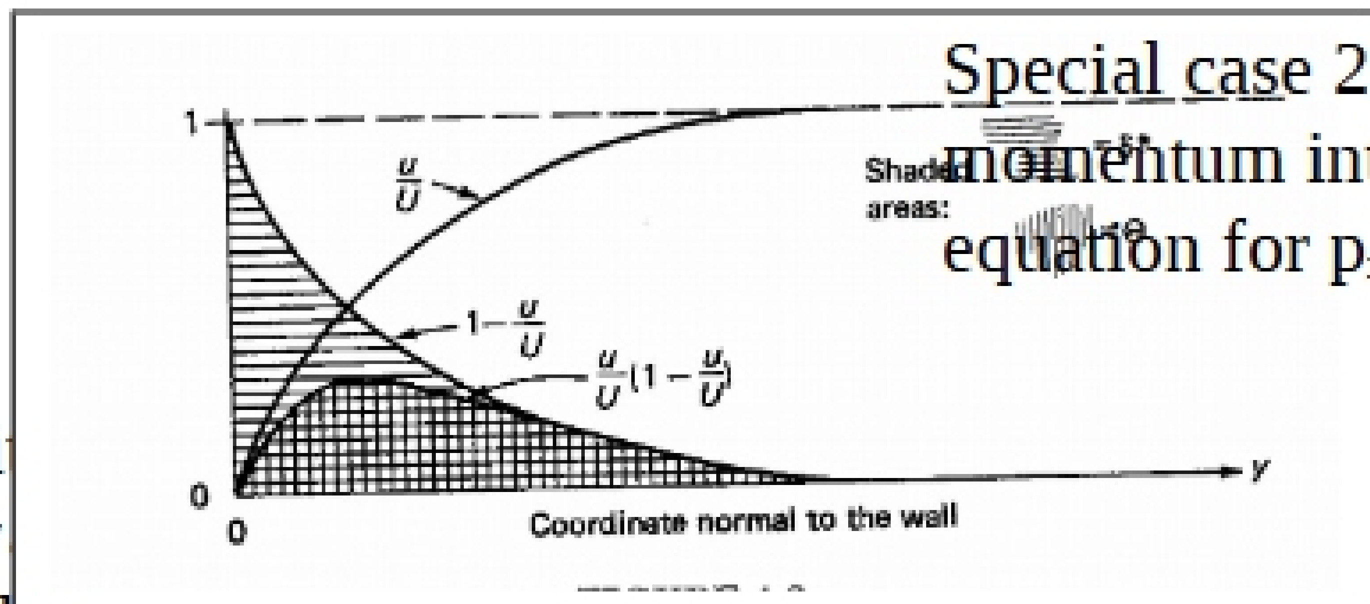
where, θ is the **momentum thickness (a function of x only)**, an important measure of the drag.

$$C_D = \frac{2D}{\rho U^2 x} = \frac{2\theta}{x} = \frac{1}{x}\int_0^x C_f dx \quad C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \Rightarrow C_f = \frac{d}{dx}(xC_D) = 2\frac{d\theta}{dx}$$

Per unit span

$$\frac{d\theta}{dx} = \frac{C_f}{2} \qquad \tau_w = \rho U^2 \frac{d\theta}{dx}$$

Special case 2D momentum integral equation for $p_x = 0$



Simple velocit

$u = U(2y/\delta - y^2/$

u(0) = 0; no slip

u(δ) = U; } matching with outer flow

$u_y(\delta)=0$;

$\delta^* = \delta/3$; $\theta = 2\delta/15$; $H = 5/2$;

$$\tau_w = 2\mu U/\delta;$$

$$\Rightarrow C_f = \frac{2\mu U/\delta}{1/2\rho U^2} = 2\frac{d\theta}{dx} = 2\frac{d}{dx}(2\delta/15);$$

$$\therefore \delta d\delta = \frac{15\mu dx}{\rho U};$$

$$\delta^2 = \frac{30\mu dx}{\rho U};$$

$$\delta/x = 5.5/\mathrm{Re}_x^{1/2}; \mathrm{Re}_x = Ux/\nu;$$

$$\delta^*/x = 1.83/\mathrm{Re}_x^{1/2};$$

$$\theta/x = 0.73/\mathrm{Re}_x^{1/2};$$

$$C_D = 1.46/\mathrm{Re}_L^{1/2} = 2C_f(L)$$

10% error, cf. Blasius

7.3. Boundary layer approximations, equations and comments